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LOCALIZED OSCILLATIONS IN A THIN FILM WITH GROWING ISLANDS[1]

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ЛОКАЛИЗОВАННЫЕ КОЛЕБАНИЯ В ТОНКИХ ПЛЕНКАХ ПРИ РОСТЕ ОСТРОВКОВ

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Рассматривается влияние динамических эффектов на рост островков на поверхности тонких пленок. Пленка моделируется слоем жидкости с инерционной свободной поверхностью с различной плотностью массы и поверхностным натяжением. Математически постановка задачи сводится к анализу системы нелинейных уравнений, описывающих эволюцию роста зародыша островка и распространение волн в пленке.

Определены условия существования локализованных собственных форм колебаний соответствующей спектральной задачи. Показано, что локализация волн в области островка приводит к увеличению скорости роста массы островка (увеличению его размера на поверхности пленки).

1. Introduction

The problems under discussion are related to localized waves near growing islands on a thin film. Before formulating the problem, a few words should be said about how we came to it and then the physical model used as the basis is to be described.

Thin film condensation and growth of films are very complex multistage processes [1]. We are interested in the stage of nucleations and separate growth of islands. That is, the span of time when on the film surface islands of new phase are nucleated and start growing. Here attention is focused on what is the driving force and the mechanism of island nucleation. We proceed from the model that the island nucleation is stipulated by transfer of the elastic stress energy of the film, which induces surface diffusion of atoms from more stressed to less stressed regions [2]. Some details of the physical model.

1. When vapor starts condensing on rigid substrate the film first flows over substrate and behaves like fluid (so-call wetting layer).
2. Materials of the substrate and the wetting layer have different lattice parameters, and the elastic energy increases with film growth.
3. When the elastic energy exceeds the wetting energy, it relaxes. One of the possible ways of relaxation is formation and growth of nuclei on the wetting layer. Due to this, the elastic energy of the film reduces.
4. The nucleation process takes some time to be completed.

For this model, the velocity of the island growth is proportional to the gradient of the elastic energy of the film U, and the evolutionary equation (see [1] and the references therein) is as follows

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{DS\nu}{k_b T_a}\nabla U, \tag{1.1}$$

where V is island volume, D is coefficient of diffusion, S is area of diffusion front, ν is atom

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volume, k_b is Boltzmann constant, T_a is temperature. The elastic energy is defined by the static stress state of the film structure, and dynamical wave processes in the wetting layer are not taken into account. This seems reasonable but only at first sight. Both the rate of nucleation and growth of islands depend on many factors (the material constituting film and substrate, temperature etc.). But in fact the rate of nucleation is much lower than the speed of wave propagation in the film. And very soon forced radiating waves decay while the islands continue to grow. If localized waves appear in the film, the situation is quite different. A localized oscillation at the special frequency does not decay with time. This can lead to additional elastic stress in the domain of islands, and may have influence on the rate of their growth. To study this problem is the aim of the present work. We are not going to describe complicated physical processes. We are only interested in qualitative results, i.e. to show that dynamical disturbances influence the island growth.

It should be noted that any type of dynamical perturbation in the film, as a rule, gives rise to both acoustic and gravitation surface waves. But having the frequency $\Omega = v/h$, where v is speed of sound and h is film thickness ($h \approx 10^{-6}$ cm), acoustic waves can not be localized near the islands of such size (the radius of island $R \approx 10^{-7}$ cm). As for gravitation waves, they are characterized by a low–frequency spectrum and, above all, are capable of being localized near the islands. The effect of gravitation is well known from a number of works on crystal growth in the outer space.

We suggest considering the wetting layer as an open waveguide having discrete inclusions with variable mass. The waveguide is simulated by a two-dimensional wetting layer of constant depth h. The layer is occupied by inviscid incompressible heavy fluid covered by an inertial free surface where islands grow. The inertial surface is an elastic membrane with variable density of mass distribution. The wetting layer is bounded from below by a flat rigid substrate. Therefore, the present work is aimed at establishing the existence of localized modes for this waveguide and to show the influence of its modes on the growth of islands.

The plan of the paper is as follows. The time-domain problem for the forced motion of the fluid layer with growing islands on the inertial surface is formulated in § 2. Mathematically, there is a non-linear problem which covers both the evolutionary equation for the growing islands and the boundary value problem for the fluid layer. The trapped mode solutions to the corresponding frequency-domain problem are given in § 3 (under the assumption that the mass of islands is a constant parameter). The method for the time-domain problem to be solved in the frame of shallow water approximation is outlined in § 4. For the constant mass of islands the large-time asymptotics of the problem is described in § 4.1, the results for the growing mass are presented and discussed in § 4.2 and § 4.3.

2. Formulation

Cartesian coordinates (x, y) are chosen with y directed vertically upwards and with the origin in the inertial surface. In the linearized time-domain problem, the fluid motion is described by the velocity potential $\Phi(x, y, t)$ that satisfies Laplace's equation

$$\nabla^2 \Phi = 0 \tag{2.2}$$

in the fluid and also the impermeability condition

$$\Phi_y = 0 \quad \text{on} \quad y = -h. \tag{2.3}$$

The free surface elevation of the fluid $\eta(x, t)$ is related to Φ through the free surface conditions

$$\begin{gathered} \eta_t = \Phi_y \quad \text{and} \\ T\eta_{xx} - (m\eta_t)_t = \rho\Phi_t + \rho g \eta + P(t)\delta(x) \\ \text{on} \quad y = 0, \end{gathered} \tag{2.4}$$

where T is coefficient of surface tension, m is membrane mass per unit length, ρ is density of fluid, $\delta(x)$ is Dirac delta–function. Generally speaking, the fluid motion may be forced by a vibration of the substrate or falling drops. In our consideration the motion is caused by the external force $P(t)$ applied at the point $(0, 0)$. The initial conditions is

$$\Phi(x, 0, 0) = \Phi_t(x, 0, 0) = 0. \tag{2.5}$$

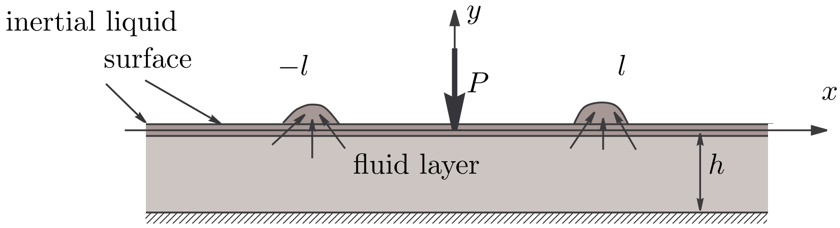

Figure 1. A schematic of islands on a film

For any fixed time the velocity potential Φ satisfies the condition at infinity

$$\nabla\Phi \to 0 \quad \text{as} \quad |x| \to \infty. \tag{2.6}$$

Let two growing spherical islands of radius R be centered at $(-l, 0)$ and $(l, 0)$. They have equal mass. The size of the growth islands is small, then m can be written as follows

$$m(x,t) = m_0 + M(t)\left[\delta(x-l) + \delta(x+l)\right], \tag{2.7}$$

where m_0 is constant initial surface mass per unit length. Assuming the island density to be not variable, we rewrite equation (1.1) so that it would define the mass of the growth island M. It can be defined through the density of elastic energy $U(x, 0, t)$ by the evolutionary equation

$$\begin{aligned} \frac{\mathrm{d}M}{\mathrm{d}t} &= \overline{D}U(x,t) \quad \text{at} \quad x = \pm l, \\ y &= 0 \quad \text{and} \quad M(0) = M_0, \end{aligned} \tag{2.8}$$

where $\overline{D} = D\pi\rho\,\nu/k_b T_a$. We split $U(x,t)$ into two terms

$$U = U_0 + U_d, \tag{2.9}$$

where U_0 is elastic energy defined by static stress state of the layer, the dynamical component U_d is a result of wave propagation in the inertial surface

$$\begin{aligned} U_0 &= \kappa\left(M_s - M\right), \\ U_d &= \frac{1}{2}T\eta_x^2 + \frac{1}{2}\rho g\eta^2 \quad \text{at} \quad x = \pm l. \end{aligned} \tag{2.10}$$

Here κ is coefficient defined by the elastic characteristics of the inertial surface and $M_s = \pi\rho R^2 h$.

Thus the motion of the film with growing inclusions is described by the time-domain problem (2.2)–(2.6) and the evolutionary equation (2.8)–(2.10). We assume that all functions are even in x.

3. Trapped modes for the frequency-domain problem

Consider the case when the island mass M is assumed to be a constant parameter. All motions are harmonic in time and have frequency ω and $\Phi(x,y,t) = \mathrm{Re}\{\varphi(x,y)\mathrm{e}^{-\mathrm{i}\omega t}\}$, $\eta(x,t) = \mathrm{Re}\{\eta(x)\mathrm{e}^{-\mathrm{i}\omega t}\}$. The function $\eta(x)$ is omitted from the free surface conditions (2.4). Then the problem for the velocity potential $\varphi(x,y)$ corresponding the problem (2.2)–(2.6) is

$$\nabla^2\varphi = 0 \tag{3.11}$$

for $-\infty < x < \infty$, $-h < y < 0$,

$$\varphi_y = 0 \quad \text{on } y = -h, \tag{3.12}$$

$$\begin{aligned} T\varphi_{xxy} - \left[\rho g - m_0\omega^2\right]\varphi_y + \rho\omega^2\varphi = \\ = -M\omega^2\left[\delta(x-l) + \delta(x+l)\right]\varphi_y \end{aligned} \tag{3.13}$$

on $y = 0$,

$$\nabla\varphi \to 0 \text{ as } |x| \to \infty. \tag{3.14}$$

It will be shown below that there exists trapped mode solution for the problem (3.11)–(3.14).

Note. In general, trapped modes are free oscillations of unbounded fluid for which the wave motion is confined to vicinity of fixed inclusions. Thus the energy of the motion is finite and there is no radiation of energy to the infinity. Such modes are non-trivial solutions of the linearized water-wave problem in the frequency domain [3]. Many publications are available where some particular structural geometries which support the trapped modes have been constructed (see [4] and the references therein), but no general proof for trapped mode existence has been found. The later also refers to the problem (3.11)–(3.14), which describes the fluid covered by the inertial surface.

Problem for the Fourier transform of the velocity potential $\varphi^F(y) = \int_{-\infty}^{+\infty} \varphi(x,y)\mathrm{e}^{-\mathrm{i}kx}\,dx$ yields

$$\frac{\mathrm{d}^2\varphi^F}{\mathrm{d}t^2} - k^2\varphi^F = 0$$

for $-h < y < 0$,

$$\begin{aligned}\left[Tk^2 + \rho g - m_0\omega^2\right]\frac{\mathrm{d}\varphi^F}{\mathrm{d}t} - \rho\omega^2\varphi^F &= \\ &= A\left[\mathrm{e}^{-kl} + \mathrm{e}^{kl}\right]M\omega^2\end{aligned}$$

for $y = 0$,

$$\frac{\mathrm{d}\varphi^F}{\mathrm{d}t} = 0 \quad \text{for } y = -h,$$

where $A(= \varphi_y(l))$ is an arbitrary constant. This has a solution

$$\begin{aligned}\varphi^F(y) &= \\ &= \frac{A\left[e^{-kl} + e^{kl}\right]}{\left[\left(Tk^2 + \rho g - m_0\omega^2\right)k\tanh kh - \rho\omega^2\right]} \times \\ &\qquad \times \frac{M\omega^2\cosh k(y+h)}{\cosh kh}.\end{aligned}$$

Applying the inverse transform, we arrive at

$$\begin{aligned}\varphi(x,y) = AM\omega^2\big[G(|x-l|,y) + \\ + G(|x+l|,y)\big], \quad (3.15)\end{aligned}$$

where

$$\begin{aligned}G(x,y) = \mathrm{i}C_0 e^{-\mathrm{i}k_0|x-\xi|}\cosh k_0(y+h) + \\ + \sum_{j=1}^{\infty} C_j e^{-k_j|x-\xi|}\cos k_j(y+h),\end{aligned}$$

$$C_0 = -\frac{2\rho g\left({c_0}^2k_0^2 - \rho^2\omega^4\right)^{1/2}}{h\left({c_0}^2k_0^2 - \rho^2\omega^4\right) + \rho\omega^2(c_0 + 2Tk_0^2)},$$

$$C_j = -\frac{2\rho g\left({c_j}^2k_j^2 + \rho^2\omega^4\right)^{1/2}}{h\left({c_j}^2k_j^2 + \rho^2\omega^4\right) + \rho\omega^2(c_j - 2Tk_j^2)},$$

$$c_0 = \rho g + T\,k_0^2 - m_0\omega^2,$$

$$c_j = \rho g - Tk_j^2 - m_0\omega^2$$

and

$$k_0,\ \pm\mathrm{i}k_1,\ \pm\mathrm{i}k_2, ...,\ \pm\mathrm{i}k_n, ...$$

is the sequence of roots of the dispersion relation

$$k\tanh kh = \frac{\rho\omega^2}{T\,k^2 - m_0\omega^2 + \rho g}.$$

The velocity potential (3.15) can split into two terms. One is outgoing progressive waves. The other behaves like standing waves which decay at the infinity. We should cancel the outgoing waves to construct trapped mode solution. From this we can have the sequence of "so-called" trapped frequencies. With these frequencies there are no outgoing waves at the infinity.

The outgoing wave is cancelled when

$$r_n = \left(n - \frac{1}{2}\right)\frac{\pi}{l}, \quad n = 1, 2, \ldots$$

and the trapped frequencies are

$$\begin{aligned}\omega_n^2 = \frac{(T\,r_n^2 + \rho g)r_n\tanh r_n h}{\rho + m_0 r_n\tanh r_n h}, \qquad (3.16)\\ 0 < \omega_1 < \omega_2 < \ldots < \omega_n < \ldots, \\ \omega_n \to \infty \text{ as } n \to \infty.\end{aligned}$$

The trapped frequencies (3.16) are point eigenvalues of the problem (3.11)–(3.14) if and only if we choose the island mass as follows

$$\begin{aligned}M_n &= \\ &= 2\rho g\left[\omega_n^2\sum_{j=1}^{\infty}\frac{\alpha_{jn}\left(1 + \mathrm{e}^{-2k_j\,l}\right)}{h\alpha_{jn} + \rho\omega_n^2\left(c_j - 2Tk_j^2\right)}\right]^{-1}, \\ &\qquad n = 1, 2, \ldots \quad (3.17)\end{aligned}$$

where $\alpha_{jn} = c_j^2\,k_j^2 + \rho^2\omega_n^4$.

Thus, each n-th trapped frequency (3.16) is supported by M_n (3.17) and

$$M_1 > M_2 > \ldots > M_n > \ldots, \quad M_n \to 0$$

as $n \to \infty$.

The trapped mode solution at ω_n has the form

$$\varphi_n(x,y,\omega_n) = AM_n\omega_n^2\begin{cases}\tilde{A} + \sum_{j=1}^{\infty}\tilde{B}_j & \text{for } |x| < l, \\ \sum_{j=1}^{\infty}\tilde{B}_j & \text{for } |x| > l,\end{cases}$$

where $\tilde{A} = (-1)^{n+1}C_0 \cos k_0 x \cosh k_0(y+h)$, $\tilde{B} = C_j \left[e^{-k_j|x-l|} + e^{-k_j|x+l|}\right] \cos k_j(y+h)$.

The coefficients C_0, C_j have been determined earlier. Thus, it is shown that there is only one trapped mode at the particular frequency (3.16) for the fixed inclusion M defined in (3.17).

Now we arrive back to the initial-value problem (2.2)–(2.6).

4. Shallow water approximation

For the shallow water approximation, potential $\overline{\Phi}(x,t)$ ($\equiv \Phi(x,0,t)$) is introduced to the problem

$$\mathbf{L}(\overline{\Phi}) = T\overline{\Phi}_{xxxx} - m_0\overline{\Phi}_{xxtt} + \frac{\rho}{h}\overline{\Phi}_{tt} - g\overline{\Phi}_{xx} = \\ = \left[M(t)(\delta(x-l)+\delta(x+l))\overline{\Phi}_{xx}\right]_{tt} + \\ + P\delta(x-\xi). \quad (4.18)$$

$$\overline{\Phi}, \overline{\Phi}_x \to 0 \text{ as } |x| \to \infty \text{ for fixed } t, \\ \overline{\Phi}(x,0) = \overline{\Phi}_t(x,0) = 0.$$

The corresponding frequency-domain problem is denoted by $\mathbf{L}_\omega(\varphi)$.

4.1. *Solution for long-duration and constant mass of the islands*

We begin with the case of the constant mass of island. For the frequency-domain problem $\mathbf{L}_\omega(\varphi)$ the obtained results (see §3) occur under the assumption $kh \ll 1$. We fix an inclusion $M = M_*$ which supports a trapped mode at the frequency $\omega_* = \omega_1$ defined in (3.16). We assume that the solution of (4.18) can be expressed by the expansion [5]

$$\overline{\Phi}(x,t) = \varphi(x,\omega_*)q(t) + \\ + \int_0^\infty \varphi_\omega(x)q_\omega(t)\mathrm{d}\omega, \quad (4.19)$$

Here $q(t)$, $q_\omega(t)$ are unknown functions and $\varphi(x,\omega_*)$ is the eigenfunction of $\mathbf{L}_\omega(\varphi)$ associated with the discrete eigenvalue $\omega_* = \omega_1$ corresponding to the particular inclusion M_* (3.17), $\varphi_\omega(x) = \varphi(x,\omega)$ is the family of symmetric eigenfunctions associated with eigenvalues ω (called radiation modes). It can be shown that the radiation modes are orthogonal in the sense of generalized functions. The trapped mode and the radiation modes satisfy the following integral identity

$$\int_{-\infty}^{+\infty} \varphi\left[\varphi_\omega - m_0 h\frac{\partial^2\varphi_\omega}{\partial x^2}\right]\mathrm{d}x = \\ = \frac{2hM_*\delta(x-l)}{\omega^2-\omega_*^2}\left[\omega^2\varphi\frac{\partial^2\varphi_\omega}{\partial x^2} - \omega_*^2\varphi_\omega\frac{\partial^2\varphi}{\partial x^2}\right]. \quad (4.20)$$

The last integral identity is the generalized condition of the orthogonality of the trapped mode to the radiation modes. Substituting (4.19) to (4.18), multiplying by $\varphi(x,\omega_*)$, integrating with respect to x and using (4.20) and dispersion relation, we arrive at

$$\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + \omega_*^2 q = Q_* P(t) \quad (4.21)$$

with initial conditions $q(0) = \mathrm{d}q(0)/\mathrm{d}t = 0$. Here

$$Q_* = \varphi(\omega_*)\left[\frac{\rho}{h}\int_{-\infty}^{+\infty}\varphi^2\mathrm{d}x - \right. \\ \left. - m_0\int_{-\infty}^{+\infty}\varphi_x^2\mathrm{d}x - 2M_*\varphi_{xx}(l)\varphi(l)\right]^{-1}.$$

An equation analogous to (4.21) can be written for $q_\omega(t)$. Then

$$q(t) = \frac{Q_*}{\omega_*}\int_0^t P(\tau)\sin\omega_*(t-\tau)\mathrm{d}\tau, \\ q_\omega(t) = \frac{Q_\omega}{\omega}\int_0^t P(\tau)\sin\omega(t-\tau)\mathrm{d}\tau,$$

and by the Riemann–Lebesque lemma it may be shown that

$$\int_0^\infty \varphi_\omega(x)\int_0^t P(\tau)\sin\omega(t-\tau)\mathrm{d}\tau\mathrm{d}\omega \to 0 \\ \text{as } t\to\infty.$$

Then for a long period of time (comparable with duration of island formation) the radiation modes decay and the solution to the problem (4.18) is reduced to the localized mode

$$\Phi(x,t) = \frac{Q_*\varphi(x,\omega_*)}{\omega_*}\int_0^t P(\tau)\sin\omega_*(t-\tau)\mathrm{d}\tau \\ \text{as } t\to\infty.$$

Thus, the force P excites the trapped mode and in the absence of friction it persists for all time. The similar results were obtained by [3]. They analyzed excitation of trapped water waves by the forced motion of a surface-piercing structure and showed that almost any forcing, whether sustained or transitory, will excite the trapped mode which does not decay at large time.

For the variable mass of the island we must consider the problem (4.18) and evolutionary equation (2.8)–(2.10) simultaneously.

4.2. *Variable mass of the island*

Let the island mass be written in the following form

$$M(t) = M_* + \mu(t), \tag{4.22}$$

where M_* is initial mass of inclusion supporting the trapped mode at frequency $\omega_* = \omega_1$ defined in (3.16). Then the problem for the potential $\overline{\Phi}(x,t)$ is

$$\begin{aligned}\mathbf{L}(\overline{\Phi}) - M_*\big[\delta(x-l)+\delta(x+l)\big]\overline{\Phi}_{xxtt} &= \\ = \mu(t)\big[\delta(x-l)+\delta(x+l)\big]\overline{\Phi}_{xxtt} &+ \\ + P(t)\delta(x-\xi)& \end{aligned} \tag{4.23}$$

$$\overline{\Phi},\ \ \overline{\Phi}_x \to 0 \quad \text{as} \quad |x| \to \infty \quad \text{for fixed} \ \ t,$$

$$\overline{\Phi}(x,0) = \overline{\Phi}_t(x,0) = 0.$$

The evolutionary equation takes the form

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = \overline{D}\left[\kappa\big(\mu_* - \mu\big) + \frac{1}{2}T\eta_x^2 + \frac{1}{2}\rho g\eta^2\right] \quad \text{at} \quad x = \pm l, \mu(0) = 0, \tag{4.24}$$

and

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = -h\overline{\Phi}_{xx}. \tag{4.25}$$

Here $\mu_* = M_s - M_*$. After introducing nondimentional time $t_1 = \sqrt{g/h}\,t$, equation (4.24) can be written as

$$\frac{\mathrm{d}\mu}{\mathrm{d}t_1} = \frac{v_d}{\sqrt{gh}}\left[\frac{h\kappa}{g}\big(\mu_* - \mu\big) + \frac{Th}{2g}\eta_x^2 + \frac{1}{2}\rho h\eta^2\right] \quad \text{at} \quad x = \pm l, \mu(0) = 0.$$

Here the coefficient $v_d = \overline{D}g$ is proportional to the growth rate of the island. The speed of wave propagation is known to significantly exceed the rate of nucleation and $\sqrt{gh} \gg v_d$ and so we can take $\varepsilon = v_d/\sqrt{gh}$ as a small parameter. This allows to assume that the trapped mode which persists for all time (in the absence of friction) can significantly affect the island growth, in which case the unknown velocity potential $\overline{\Phi}$ can be found, using only the first term in the expansion (4.19)

$$\overline{\Phi}(x,t_1) = \overline{\varphi}(x,\omega_*)q(t_1).$$

Bellow nondimantional time t_1 will be written without subscript. Consider a simple example of the external force $P(t) = P\delta(t)$ Thus the problem (4.23)–(4.25) is reduced to

$$[1 - \varepsilon L\mu]\,\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + \overline{\omega}_*^2 q = 0, \tag{4.26}$$

$$\frac{\mathrm{d}\mu}{\mathrm{d}t} = \varepsilon\left[\frac{\kappa h}{g}\big(\mu_* - \mu\big) + \frac{Th}{2g}\,\eta_x^2 + \frac{1}{2}\rho h\eta^2\right] \quad \text{at } x = \pm l, \tag{4.27}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}t} = -c\,\varphi_{xx}q. \tag{4.28}$$

Here $\overline{\omega}_*^2 = \omega_*^2 h/g$, $c = \sqrt{gh}$,

$$\varepsilon L = \frac{2[\overline{\varphi}_x(l)]^2}{\Delta_*}.$$

where

$$\Delta_* = \frac{\rho}{h}\int_{-\infty}^{+\infty}[\overline{\varphi}]^2\mathrm{d}x + m_0\int_{-\infty}^{+\infty}[\overline{\varphi}_x]^2\mathrm{d}x + 2M_*[\overline{\varphi}_x(l)]^2.$$

The initial conditions are

$$q(0) = 0, \quad \frac{\mathrm{d}q(0)}{\mathrm{d}t} = F,$$
$$\mu(0) = 0,$$

where $F = Ph/g$. We add the small parameter ε to equation (4.26). This, as can be shown, is quite correct.

Then, using the multiple scale method we take two scale of time t and $\tau = \varepsilon t$ and apply expressions of unknown functions in the small parameter ε

$$q(t,\tau) = q_0 + \varepsilon q_1 + \dots,$$

$$\mu(t,\tau) = \mu_0 + \varepsilon\mu_1 + \dots,$$

$$\eta(t,\tau) = \eta_0 + \varepsilon\eta_1 + \dots$$

Let us restrict further consideration to two approximations. For the zero approximation we get

$$\frac{\partial^2 q_0}{\partial t^2} + \overline{\omega}_*^2 q_0 = 0, \quad q_0(0) = 0, \quad \frac{\mathrm{d}q_0(0)}{\mathrm{d}t} = F,$$

$$\frac{\partial \mu_0}{\partial t} = 0, \quad \mu_0(0) = 0,$$

$$\frac{\partial \eta_0}{\partial t} = -c\varphi_{xx} q_0.$$

Here μ_0 is independent of t and the first approximation has the form

$$\frac{\partial^2 q_1}{\partial t^2} + \overline{\omega}_*^2 q_1 = L\mu_0 \frac{\partial^2 q_0}{\partial t^2} - 2\frac{\partial^2 q_0}{\partial t \partial \tau},$$

$$\frac{\partial q_1(0)}{\partial t} = \frac{\partial q_0(0)}{\partial \tau}, \tag{4.29}$$

$$\frac{\partial \mu_1}{\partial t} = -\frac{\partial \mu_0}{\partial \tau} + \kappa_g(\mu_* - \mu) + \\ + \frac{1}{2} T_g \eta_{0x}^2 + \frac{1}{2}\rho h \eta_0^2 \tag{4.30}$$

at $x = \pm l$,

$$\frac{\partial \eta_1}{\partial t} = -\frac{\partial \eta_0}{\partial \tau} - c\,\varphi_{xx} q_1. \tag{4.31}$$

Here $\kappa_g = \kappa h/g$, $T_g = Th/g$. Then further application of the multiple scale method yields that

$$\mu_0 = \big(\mu_* + \beta\big)\beta_*, \tag{4.32}$$

here $\beta_* = 1 - \exp\left[-\kappa_g \tau\right]$,

$$q_0(t,\tau) = \\ = \frac{F}{\overline{\omega}_*} \sin \overline{\omega}_* \left[t + \frac{L\big(\mu_* + \beta\big)}{2\kappa_g} \Big(\beta_* + \kappa_g \tau\Big) \right] \tag{4.33}$$

and

$$\mu_1 = \frac{\beta \kappa_g}{2\overline{\omega}_*} \sin 2\overline{\omega}_* \left[t + \frac{L(\mu_* + \beta)}{2\kappa_g} \Big(\beta_* + \kappa_g \tau\Big) \right].$$

Here $\beta = \dfrac{hc^2F^2}{8g\kappa_g\overline{\omega}_*^4}\left[T_g \varphi_{xxx}^2(l) + \rho h \varphi_{xx}^2(l)\right]$.

4.3. Influence of localization waves on the island growth

The island mass $M(t)$ then follows from (4.22) and, after some manipulation, it is found that

$$M(t) = M_s + (M_* - M_s)\exp[-\varepsilon\kappa_g t] + \\ + \beta\Bigg\{ 1 - \exp[-\varepsilon\kappa_g t] + \\ + \frac{\varepsilon\kappa_g}{2\overline{\omega}_*}\sin 2\overline{\omega}_* \left[t + \frac{L(M_s - M_* + \beta)}{2\kappa_g} \times \\ \times \Big(1 + \varepsilon\kappa_g t - \exp[-\varepsilon\kappa_g t]\Big)\right]\Bigg\} + O(\varepsilon^2).$$

The last expression describes the island growth qualitatively. The first terms correspond to the island growth caused by the static stress only. The influence on the island growth exerted by the trapped wave is expressed by the third term, with the constant β as a factor.

Thus, the existence of localized wave on the film surface leads to the increase in the rate of island growth. When determining the size of growing islands, this effect should be taken into account.

5. Conclusion

A thin film with growing islands has been modelled as the two–dimensional non–stationary problem for a fluid layer with the inertial free surface on which there are two growing mass inclusions. For the corresponding frequency–domain problem there was obtained a trapped mode solution. It was shown that in the time domain problem perturbation force will excite the localized wave near the islands and in the absence of friction the wave will persist for all time. This creates additional stress in the film and leads to the increase in the rate of island growth.

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