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DIRECTED ANTENNA IN BLOCK STRUCTURE

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Abstract. The work is devoted to studying the problem of creating directional antennas in block structures, blocks of which are heterogeneous interacting bodies.

Previously, the authors have conducted a similar study for the case of multi-layer medium on an elastic half-space. The area of the study was a half-space in which the antenna direction was formed. Contact between the layers and a half-space made it possible for the vibration influence directed from the surface reach half-space. The problem is much more complicated in the case of an arbitrary block structure. Here it is necessary to solve the problem of contact interaction of blocks that require a specification of regions occupied by blocks and the nature of the contact between them. The study is based on earlier results obtained by the method of block element, and the most recent results based on topological methods for solving boundary value problems. Further research related to the question of localization of the stress-strain state in a separate block, which is also broad enough task, but which has already been performed separately.

Keywords: block element, factorization, topology, integral and differential factorization methods, exterior forms, block structures, boundary problems

Introduction

In this study, the differential factorization method, which has been applied in [1, 2] to an individual convex isotropic elastic body, is extended to the case of block structures, in particular, layered structures. As was noted in [3], this circumstance opens up the possibility to investigate boundary-value problems for differential equations with variable coefficients, as well as nonlinear boundary-value problems.

The formulas derived in [1, 2] for an individual convex elastic body represented an approximate solution describing its stress-strained state. This approximate solution becomes more accurate as the shape of the body approaches a half-space. The approximate equation thus constructed can be refined also by inverting the systems of integral equations presented in the cited works. Similar results can be obtained for block structures [4] but, as is shown below, the large number of blocks and the variety of possible combinations lead to much more complex relationships.

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The differential factorization method described in [1, 2] as applied to an individual domain is extended herein to a collection of neighbouring domains, which are referred to as block structures. As applied to boundary value problems for such collections of domains, this method has specificity features that distinguish it from traditional approaches. For example, boundary conditions in the differential factorization method cannot be satisfied in the traditional form by introducing the limiting values of solutions and their derivatives on the boundary. The cause id that the derivatives of the solution constructed by the method on the boundary have mot only classical components but also components in the form of generalizes functions, namely, δ-delta functions and their derivatives [1, 2]. Their origin is explained in detail in [1, 2], and they are not an obstruction to solving boundary value problems. In this paper, we show how to overcome these difficulties when the differential factorization method is applied to block structures.

1. The block element method

By block structures, we mean materials occupying bounded, semibounded, or unbounded domains, which are called contacting blocks. It is assumed that each block in a block structure has its own specific behavioral in response to physical fields of a various nature. It is also assumed that these fields are described by boundary value problems for systems of coupled partial differential equations with constant coefficients. Media of this type are typical of the earth's crust , structural materials under complex physical-mechanical conditions [4], nonmaterials, crystal structures of various arrangements, and electronics materials. A similar structure is also posses by various materials, including those created by combining only nanoscale components or macro- and nanoscale components.

We consider structures with three-dimensional blocks. The absence of considerable constraints on boundary value problems describing the properties of individual blocks suggests that these block structures can have a wide variety of properties. In the general case, the concept of a block requires that the boundary of the domain a boundary value problem, including multiply connected domains, be unchanged and piecewise smooth. Each block can be bounded or unbounded and can involve coupled processes related to solid and fluid mechanics and electromagnetic, diffusion, thermal, acoustic, and other processes. Block structures are more general objects than piecewise homogeneous structures, in which the physical parameters of the medium are assumed to change in jumps in the transition from one block to another with the preservation of the medium material. The last property means that certain coefficients in the differential equations of a boundary value problem undergo jump variations in the transition from one block to another with the type of the boundary value problem being preserved.

Block structures have a wider range of properties than piecewise homogenous structures. This follows from the variety of blocks' properties, their shapes, and the character of interblock interactions and also results from the interaction of physical fields, some of which are produced or transformed by blocks. A special case of block structures is layered structures. Such structures with plane boundaries for linear boundary value problems can be viewed as fairly thoroughly investigated. Block structures are studied primarily by numerical methods, for which unbounded domains always present difficulties. The differential factorization method, which is a generalization of the integral transform method, gives answers to questions concerning the properties of physical fields in each block even at the stage of solving boundary value problems.

Note that integral transforms in a boundary value problem for partial differential equations in a domain Ω are a convenient research tool when the differential equations, Ω, and the functions describing an integral transform are consistent. By consistency, we mean the possibility of transforming partial differential equations into ordinary ones by applying an integral transform and the setting of boundary conditions on the boundary described by constant geometric parameter values. This property holds if the integral-transform functions are the eigenfunctions of the differential operator in Ω. In terms of topological algebra, this property holds if the transformation groups generated by an automorphism of the manifold Ω have representations that are invariant under a differentiable mapping of the vector field defined on this manifold. For several simple domains, which are referred to as classical, these are the Fourier transform in domains with plane boundaries, the Bessel transform in

domains with circular boundaries, the Bessel-Legendre transform in domains with spherical boundaries, which are applied, for example, to the Helmholtz, Schrodinger, Lame, and Navier-Strokes equations with constant coefficients.

It was shown that these and other integral transforms are consequences of self-mappings of manifolds generating transformation groups of space and their motions. Representations of these groups are obtained by introducing the special functions mentioned above. In the case of classical domains, boundary value problems are relatively easy to solve. Specifically, after applying an integral transform, they are reduced to simple functional or ordinary differential equations and then integral inversion is used.

For boundary value problems in domains of complex geometry, we use the differential factorization method, which reduce them to functional equations with dimension reduction.

We formulate the following boundary value problem for a block structure. Assume that the block-structure domain Ω consists of subdomains Ω_b, $b = 1, 2, \ldots, B$ with boundaries $\partial\Omega_b$ It may happen that a portion of the block's boundary is shared with another block, in which case it is a contact boundary. The remaining non-contact portion can be free or subject to external forces. It is assumed that a boundary value problem for systems of partial differential equations with (their own) constant coefficients is set in each domain Ω_b.

For each block, the boundary value problem for the system of P partial differential equations in the three-dimensional block domain Ω can be written as

$$\mathbf{K}_b(\partial x_1, \partial x_2, \partial x_3)\varphi_b = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{p=1}^{P} A^b_{spmnk}\varphi_{bp,x_1^{(m)}x_2^{(n)}x_3^{(k)}} = 0,$$

$$s = 1, 2, \ldots, P_b, \quad A^b_{sqmnk} = \text{const},$$

$$\varphi_b = \{\varphi_{b1}, \varphi_{b2}, \ldots, \varphi_{bP}\}, \quad b = 1, 2, \ldots, B.$$

$$\varphi = \{\varphi_s\}, \quad \varphi(\mathbf{x}) = \varphi(x_1, x_2, x_3), \quad \mathbf{x} = \{x_1, x_2, x_3\}, \quad \mathbf{x} \in \Omega_b \tag{1.1}$$

The following matching conditions are set on the common contact boundary $\partial\Omega_b \cap \partial\Omega_d$

$$\mathbf{R}_b(\partial x_1, \partial x_2, \partial x_3)\varphi_b + \mathbf{R}_d(\partial x_1, \partial x_2, \partial x_3)\varphi_d = \sum_{m=1}^{M_1}\sum_{n=1}^{N_1}\sum_{k=1}^{K_1}\sum_{p=1}^{P}\left[B^b_{spmnk}\varphi_{bp,x_1^{(m)}x_2^{(n)}x_3^{(k)}} + B^d_{spmnk}\varphi_{dp,x_1^{(m)}x_2^{(n)}x_3^{(k)}}\right] = f_{bds}, \tag{1.2}$$

$$s = 1, 2, \ldots, s_{b0} < P, \quad \mathbf{x} \in \partial\Omega_b \cap \partial\Omega_d,$$
$$M_1 < M, \quad N_1 < N, \quad K_1 < K,$$
$$b, d = 1, 2, \ldots, B.$$

The boundary value problem is studied in the spaces of tempered distributions described in [1].

In the general form, the above boundary conditions describe the contact of blocks with the relevant components of physical fields coinciding on the common boundaries as dedicated by the corresponding physical laws. In particular, conditions (1.2) can be significantly simpler. For example, they can lack boundary external forces and express only the equality between the solutions and their derivatives on a common boundary. However, as mentioned above, the derivatives of solutions written in integral form cannot be equated, since their components in the factorization method are generalized functions [2]. On noncontact boundaries, we set the boundary conditions of the boundary value problem considered in [1]. The scheme for applying the differential factorization method to such domains can be described as follows.

Following the differential factorization method [1], the boundary value problem is reduced to a system of functional equations with each domain Ω_b considered separately. As a result, we obtain the system of functional equations.

$$\mathbf{K}_b(\alpha)\boldsymbol{\varphi}_b = \iint\limits_{\partial\Omega_b} \boldsymbol{\omega}_b,$$

$$\mathbf{K}_b(\alpha) \equiv -\mathbf{K}_b(-i\alpha_1, -i\alpha_2, -i\alpha_3) = \|k_{bnm}(\alpha)\|, \tag{1.3}$$

$$b = 1, 2, \ldots, B.$$

Here, we used the notation adopted in [1] with additional indices b. For example, $\boldsymbol{\omega}_b$ is the vector of exterior forms of the boundary value problem in Ω_b.

Comparing this case with that considered in [1], we note that boundary conditions (1.2) generally contain the values of the solutions and their derivatives on the boundary at least in two neighboring domains. This is a substantial difference of block structures from objects analyzed in [1].

According to the differential factorization method, the next step consists of factorizing the matrix function $\mathbf{K}_b(\alpha)$ given by (1.3). For this purpose, we choose a matrix function $\mathbf{K}_b^*(\alpha_3^\nu, m)$ of order $P-1$ obtained by deleting the row and column indexed by m in the adjoint matrix function $\mathbf{K}_b^*(\alpha_3^\nu)$ such that the zeros ξ_n^ν of its determinant $Q_b\left(\alpha_3^\nu\right) = \det \mathbf{K}_b\left(\alpha_3^\nu, m\right)$ do not coincide with the zeros z_{s+}^v, z_{s-}^v [1].

The elements of the inverse matrix function are denoted by

$$[\mathbf{K}_b^*(\alpha_3^\nu, m)]^{-1} = \left\|Q_b^{-1} Q_{psb}\right\|$$

Then the elements of $\mathbf{K}^{-1}\left(\alpha_3^\nu, -\right)$ given by

$$\mathbf{K}_b^{-1}\left(\alpha_{3,}^v -\right) = \begin{pmatrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & \ddots & & & \\ S_{m1} & S_{m2} & \dots & S_{mm} & \dots & S_{mN} \\ & & & & \ddots & \\ 0 & & & & & 1 \end{pmatrix} \tag{1.4}$$

can be represented in integral the form

$$S_{mp}(\alpha_3^\nu) = = \frac{1}{2\pi i} \oint\limits_{\Gamma_\mp} {\sum_{s=1}^N}' \frac{Q_{psb}(u_3) M_{sm}(u_3) du_3}{Q_b(u_3) K(u_3)(u_3 - \alpha_3^\nu)} - - \left(\frac{1}{2} \mp \frac{1}{2}\right) \frac{R_{mp}(\alpha_3^\nu)}{K(\alpha_3^\nu)}, \quad m \neq p,$$

$$\frac{R_{mp}(\alpha_3^\nu)}{K_b(\alpha_3^\nu)} = \frac{Z_{mp}(\alpha_3^\nu)}{Q_b(\alpha_3^\nu) K_b(\alpha_3^\nu)} + + \sum_n \frac{Z_{mp}(\xi_n^\nu)}{Q_b'(\xi_n^\nu) K_b(\xi_n^\nu)(\xi_n^\nu - \alpha_3^\nu)}$$

$$S_{mm}(\alpha_3^\nu) = K_b^{-1}(\alpha_3^\nu), \quad \alpha_3^\nu \in \lambda_\mp$$

$$Z_{mp}(\alpha_3^\nu) = {\sum_{s=1}^N}' Q_{psb}(\alpha_3^\nu) M_{sm}(\alpha_3^\nu)$$

Here, Γ_+ is a closed contour such that the domain λ_+ contains only the zeros z_{s+}^v, z_{s-}^v and, while the domainλ_- contains only the zeros ξ_n^ν. The closed contour Γ_- encloses a domain containing all the zeros z_{s+}^v, z_{s-}^v, and ξ_n^ν. Representation (1.4) implies that the elements of $\mathbf{K}_b^{-1}\left(\alpha_3^\nu, -\right)$ are rational functions with their only singularities being z_{s+}^v, z_{s-}^v . The term $K_b^{-1}\left(\alpha_3^\nu\right)$ containing them is given explicitly.

In the case of noncontact boundaries, the boundary conditions in the differential factorization method are set according to the rules described in [1].

The boundary conditions are fulfilled according to the following scheme. First boundary conditions on the noncontact boundary of each block are taken to the corresponding vectors of exterior forms in functional equations (1.3). For contact blocks, matching conditions (1.2) hold on the common boundaries of neighboring blocks. Depending on the properties of the described fields, these conditions can include some relations for the solutions and their derivatives. In the simplest case, this is the equality of the solutions and their derivatives on the common boundary in the transition from one block to another. These relations are taken to the corresponding vectors of exterior forms of functional equations (1.3), which are preliminary solved for the unknown normal derivatives on the boundary. The last procedure ensures the fulfillment of contact boundary conditions (1.2) in the solution to pseudodifferential equations, which can be proved following the scheme described in [2].

Assume that the blocks are convex. Omitting the intermediate transformations, which can be found in [1], we find that the solution in each block is represented as

$$\varphi_b(\mathbf{x}^\nu) = \frac{1}{8\pi^3} \iiint\limits_{-\infty}^{\infty} \mathbf{K}_{rb}^{-1}\left(\alpha_3^\nu\right) \mathrm{K}_b^{-1}\left(\alpha_3^\nu, -\right) \times \times \iint\limits_{\partial\Omega} \boldsymbol{\omega}_b e^{-i\left\langle \alpha_3^\nu x_3^\nu \right\rangle} d\alpha_1^\nu d\alpha_2^\nu d\alpha_3^\nu,$$

$$\mathbf{x}^\nu \in \Omega_b.$$

To illustrate this solution, we evaluate the integral with respect to α_3^ν by applying Leray's residue form theory to obtain

$$\varphi_b(\mathbf{x}^v) = \frac{1}{4\pi^2}\iint\limits_{-\infty}^{\infty}\sum_s e^{-i\left(\alpha_1^v x_1^v+\alpha_2^v x_2^v\right)}\times$$

$$\times\left[\mathbf{K}_{rb}^{-1}\left(i\frac{\partial}{\partial x_3^v}\right)\mathbf{T}_{+b}\left(\alpha_1^v,\alpha_2^v,z_{s+}^v\right)e^{-iz_{s+}^v x_3^v}-\right.$$

$$\left.-\mathbf{K}_{rb}^{-1}\left(i\frac{\partial}{\partial x_3^v}\right)\mathbf{T}_{-b}\left(\alpha_1^v,\alpha_2^v,z_{s-}^v\right)e^{-iz_{s-}^v x_3^v}\right]$$
$$d\alpha_1^\nu d\alpha_2^\nu$$

Here, the boundary $\partial\Omega_b$ for the chosen $x_3^\nu<0$, $\mathbf{x}^\nu\in\Omega$ is divided as follows:

$$\iint\limits_{\partial\Omega_b}\boldsymbol{\omega}_b=\iint\limits_{\partial\Omega_{+b}}\boldsymbol{\omega}_b+\iint\limits_{\partial\Omega_{-b}}\boldsymbol{\omega}_b,$$

$$\iint\limits_{\partial\Omega_{+b}}\boldsymbol{\omega}_b\exp(-i\alpha_3^\nu x_3^\nu)\to 0,\quad \operatorname{Im}\alpha_3^\nu\to\infty,$$

$$\iint\limits_{\partial\Omega_{-b}}\boldsymbol{\omega}_b\exp(-i\alpha_3^\nu x_3^\nu)\to 0,\quad \operatorname{Im}\alpha_3^\nu\to-\infty.$$

If a block degenerates into a half-space or a layered medium, the pseudodifferential equations appearing in the course of solving the boundary value problem degenerate into algebraic equations. The latter are inversed, and the solution is constructed in a finite form [1].

If the block under study is not a convex body, the boundary value problem is analyzed by the generalized factorization method.

2. The block structure

Let us assume that a domain Ω of a block structure consists of contracting convex domains Ω_b, $b=1,2,\ldots,B$ with boundaries $\partial\Omega_b$. It may happen that a portion f the boundary $\partial\Omega_{bd}$of a certain b block coincides with the boundary of another d block $d=1,2,\ldots,B$. Such a portion is called contracting. The remaining portions of the boundaries of both domains are noncontracting and will be denoted below by subscripts with one letter: $\partial\Omega_b$, $\partial\Omega_d$. These boundaries can be free of or subjected to external actions. It is assumed that, in each domain $\Omega_b$one of the boundary-value problems considered in [1,2] is formulated in terms of the systems of differential equations with partial derivatives, the constant coefficients of which are different in each domain.

For each block $b=1,2,\ldots,B$ characterized by its own mechanical characteristics, the equations of the isotropic elasticity theory can be written in the following form [1,2]:

$$\begin{aligned}&(\lambda_b+\mu_b)\operatorname{graddiv}\mathbf{u}_b+\mu\Delta\mathbf{u}_b-\delta_b\mathbf{u}_b=0,\\&\mathbf{u}_b=\{u_{b1},u_{b2},u_{b3}\}\end{aligned}\tag{2.1}$$

where the notation is the same as in the papers cited. On the noncontracting portions of the boundary, traditional boundary conditions of the elasticity theory are set [1, 2]. In the contracting parts, in particular, on $\partial\Omega_{bd}$, the conditions of equality of the stress vectors are formulated as follows:

$$\begin{aligned}\mathbf{u}_b=\mathbf{u}_d,\quad \mathbf{u}_c=\{u_{c1},u_{c2},u_{c3}\}\\ \mathbf{t}_b=\mathbf{t}_d,\quad \mathbf{t}_c=\{t_{c1},t_{c2},t_{c3}\},\\ t_{c1}=\sigma_{c13},\quad t_{c2}=\sigma_{c23},\quad t_{c3}=\sigma_{c33}\end{aligned}\tag{2.2}$$

Using the differential factorization method [1, 2], we reduce the boundary-value problem to the system of functional equations, considering each domain Ω_b individually. As a result, we obtain the following system of functional equations:

$$\mathbf{K}_b(\alpha)\boldsymbol{\varphi}_b=\iint\limits_{\partial\Omega_b}\boldsymbol{\omega}_b,$$

$$\begin{aligned}\mathbf{K}_b(\alpha)\equiv-\mathbf{K}_b(-i\alpha_1,-i\alpha_2,-i\alpha_3)=\\=\|k_{bnm}(\alpha)\|,\\ b=1,2,\ldots,B.\end{aligned}\tag{2.3}$$

where the notation is the same as in [1,2] with the addition of subscripts b. In particular, $\boldsymbol{\omega}_b$ is the vector of external forms of the boundary-value problem in domain Ω_b.

Comparing this case with those considered in [1, 2], it should be noted that boundary conditions (1.2) generally contain stresses and displacements on the boundary from at least two neighboring domains. In this respect, the block structures significantly differ from the individual bodies studied in [1, 2]. According to the algorithm of the differential factorization method, the boundary conditions for noncontracting boundaries are applied according to the rules stipulated in [3].

Without repeating the application of algorithms described in [1, 2], we will present here the final form of pseudodifferential equations

for the case of contact between two bodies. Fulfillment of the boundary conditions is ensured as follows. First, the boundary conditions on the noncontracting boundary for each individual block are introduced in the corresponding vectors of exterior forms of functional equations (2.3). When blocks are in contact, matching conditions (2.2) on the coinciding boundaries of the neighboring blocks $\partial\Omega_{bd}$ are valid. These relationships should be introduced in the corresponding vector of exterior forms only of one of the functional equations, whereas the vector of exterior forms of the second equation remains unchanged. As was proved in [4], this procedure ensures fulfillment for the boundary conditions (2.2) and does not require separation of the generalized functions from classical components appearing in a natural way in solutions obtained using the factorization method.

Having omitted the procedure of applying the differential factorization method to the boundary-value problem under consideration, including its realization in each domain Ω_b and on Ω_d as was performed in [1,2], we present the pseudodifferential equations for a block structure consisting of two blocks:

$$\begin{aligned}\mathbf{M}_c^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu},\alpha_{3r-}^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu}))\times\\ \times\mathbf{U}_c^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu},\alpha_{3r-}^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu}))-\\ -\mathbf{D}_c^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu},\alpha_{3r-}^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu}))\times\\ \times\mathbf{T}_c^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu},\alpha_{3r-}^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu}))+\\ +\sum_{\tau=1}^{\mathbf{T}}{}'\Big[\mathbf{M}_c^{\tau}(\alpha_1^{\nu},\alpha_2^{\nu},\alpha_{3r-}^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu}))\times\\ \times\mathbf{U}_c^{\tau}(\alpha_1^{\nu},\alpha_2^{\nu},\alpha_{3r-}^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu}))-\\ -\mathbf{D}_c^{\tau}(\alpha_1^{\nu},\alpha_2^{\nu},\alpha_{3r-}^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu}))\times\\ \times\mathbf{T}_c^{\tau}(\alpha_1^{\nu},\alpha_2^{\nu},\alpha_{3r-}^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu}))\Big]=0\end{aligned}\quad(2.4)$$

Here, $c=b$ in the case of domain Ω_b and $c=d$ in the case of domain Ω_d.

Applying the methods described in [2] and retaining the notation used in that study, these pseudodifferential equations can be reduced to systems of integral equations. The system of integral equations for domain Ω_b, written with respect to vector $t\mathbf{t}_b^{\nu}$, $\mathbf{t}_d^{\nu}$ for the displacement vectors $\mathbf{u}_b^{\nu}$, $\mathbf{u}_d^{\nu}$ set on noncontracting boundaries, has the following form:

$$\begin{aligned}\iint\limits_{\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}}\mathbf{k}_b^{\nu}(x_1^{\nu}-\xi_1^{\nu},x_2^{\nu}-\xi_2^{\nu})\mathbf{t}_b^{\nu}(\xi_1^{\nu},\xi_2^{\nu})d\xi_1^{\nu}d\xi_2^{\nu}+\\ +\sum_{\boldsymbol{\tau}=1}^{T}{}'\iint\limits_{\partial\boldsymbol{\Omega}_{b\boldsymbol{\tau}}}\mathbf{k}_b^{\nu\tau}(x_1^{\nu},\xi_1^{\tau},x_2^{\nu},\xi_2^{\tau})\mathbf{t}_b^{\tau}(\xi_1^{\tau},\xi_2^{\tau})d\xi_1^{\tau}d\xi_2^{\tau}=\\ =\mathbf{u}_b^{\nu}(x_1^{\nu},x_2^{\nu})+\\ +\sum_{\boldsymbol{\tau}=1}^{T}{}'\iint\limits_{\partial\boldsymbol{\Omega}_{b\boldsymbol{\tau}}}\mathbf{b}_b^{\nu\tau}(x_1^{\nu},\xi_1^{\tau},x_2^{\nu},\xi_2^{\tau})\mathbf{u}_b^{\tau}(\xi_1^{\tau},\xi_2^{\tau})d\xi_1^{\tau}\xi_2^{\tau},\end{aligned}\quad(2.5)$$

$$x_1^{\nu},x_2^{\nu}\in\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}};\quad 1\leqslant\nu\leqslant T.$$

For the domain Ω_d contacting with the domain Ω_b along the boundary $\partial\Omega_{bd}$, the system of integral equations with allowance for boundary conditions (1.2) takes the following form:

$$\begin{aligned}\iint\limits_{\partial\boldsymbol{\Omega}_{p\boldsymbol{\nu}}}\mathbf{k}_d^{\nu}(x_1^{\nu}-\xi_1^{\nu},x_2^{\nu}-\xi_2^{\nu})\mathbf{t}_c^{\nu}(\xi_1^{\nu},\xi_2^{\nu})d\xi_1^{\nu}d\xi_2^{\nu}+\\ +\sum_{\boldsymbol{\tau}=1}^{T_1}{}'(x_1^{\nu},\xi_1^{\tau},x_2^{\nu},\xi_2^{\tau})\mathbf{t}_d^{\tau}(\xi_1^{\tau},\xi_2^{\tau})d\xi_1^{\tau}d\xi_2^{\tau}+\\ +\sum_{\boldsymbol{\tau}=1}^{T_2}{}'(x_1^{\nu},\xi_1^{\tau},x_2^{\nu},\xi_2^{\tau})\mathbf{t}_b^{\tau}(\xi_1^{\tau},\xi_2^{\tau})d\xi_1^{\tau}d\xi_2^{\tau}=\\ =\mathbf{u}_c^{\nu}(x_1^{\nu},x_2^{\nu})+\\ +\sum_{\boldsymbol{\tau}=1}^{T_1}{}'(x_1^{\nu},\xi_1^{\tau},x_2^{\nu},\xi_2^{\tau})\mathbf{u}_d^{\tau}(\xi_1^{\tau},\xi_2^{\tau})d\xi_1^{\tau}\xi_2^{\tau}\\ +\sum_{\tau=\mathbf{1}}^{T_2}{}'(x_1^{\nu},\xi_1^{\tau},x_2^{\nu},\xi_2^{\tau})\mathbf{u}_{\mathbf{b}}^{\tau}(\xi_{\mathbf{1}}^{\tau},\xi_{\mathbf{2}}^{\tau})\mathbf{d}\xi_{\mathbf{1}}^{\tau}\xi_{\mathbf{2}}^{\tau},\end{aligned}\quad(2.6)$$

$$\begin{aligned}x_1^{\nu},x_2^{\nu}\in\partial\boldsymbol{\Omega}_{d\boldsymbol{\nu}};\quad 1\leqslant\nu\leqslant T=T_1+T_2;\\ x_1^{\nu},x_2^{\nu}\in\partial\boldsymbol{\Omega}_{d\boldsymbol{\nu}};\quad 1\leqslant\nu\leqslant T=T_1+T_2.\end{aligned}$$

Here, $c=d$, $p=d$, if x_1^{ν}, $x_2^{\nu}\in\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}$; and $c=b$, $p=bd$, if x_1^{ν}, $x_2^{\nu}\in\partial\boldsymbol{\Omega}_{bd\boldsymbol{\nu}}$, T_1 and T_2 are the numbers of unity partition of the noncontracting portion of the boundary $\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}$and the contracting portion $\partial\boldsymbol{\Omega}_{bd\boldsymbol{\nu}}$, respectively; and the primed sum symbols imply that the terms with $\nu=\tau$ in these sums are missing if they are present in the same sum symbol. The kernels of the integral equations are as follows (in the notation from [2]):

$$\mathbf{K}_c^{\nu}(\alpha_1^{\nu},\alpha_2^{\nu})=(\mathbf{M}_c^{\nu})^{-1}\mathbf{D}_c^{\nu},$$

$$\mathbf{K}^{\boldsymbol{\nu\tau}}(\alpha_1^\nu,\alpha_2^\nu)=(\mathbf{M}_c^\nu)^{-1}\mathbf{D}_c^\tau,$$

$$\mathbf{B}_c^{\nu\tau}(\alpha_1^\nu,\alpha_2^\nu)=(\mathbf{M}_c^\nu)^{-1}\mathbf{M}_c^\tau.$$

$$\mathbf{k}_c^\nu(x_1^\nu,x_2^\nu)=\mathbf{F}_2^{-1}\mathbf{K}_c^\nu(\alpha_1^\nu,\alpha_2^\nu);$$

$$\mathbf{k}_c^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)=\mathbf{F}_2^{-1}\mathbf{K}_c^{\nu\tau}(\alpha_1^\nu,\alpha_2^\nu)\exp i\langle\mathbf{c}_\tau^\nu\boldsymbol{\alpha}^\nu,\boldsymbol{\xi}^\tau\rangle$$

$$\mathbf{b}_c^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)=\mathbf{F}_2^{-1}\mathbf{B}_c^{\nu\tau}(\alpha_1^\nu,\alpha_2^\nu)\exp i\langle\mathbf{c}_\tau^\nu\boldsymbol{\alpha}^\nu,\boldsymbol{\xi}^\tau\rangle; \quad (2.7)$$

$$\mathbf{t}_c^\nu(x_1^\nu,x_2^\nu)=\mathbf{F}_2^{-1}\mathbf{T}_c^\nu(\alpha_1^\nu,\alpha_2^\nu);$$

$$\mathbf{u}_c^\nu(x_1^\nu,x_2^\nu)=\mathbf{F}_2^{-1}\mathbf{U}_c^\nu(\alpha_1^\nu,\alpha_2^\nu).$$

The formulas take place for $c=b$ and $c=d$.

If the stress vectors $\mathbf{t}_b^\nu$, $\mathbf{t}_d^\nu$ are given on the boundary, the corresponding system of equations with respect to the displacement vectors $\mathbf{u}_b^\nu$, $\mathbf{u}_d^\nu$ takes the following form:

$$\iint\limits_{\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}}\mathbf{n}_b^\nu(x_1^\nu-\xi_1^\nu,x_2^\nu-\xi_2^\nu)\mathbf{u}_b^\nu(\xi_1^\nu,\xi_2^\nu)d\xi_1^\nu d\xi_2^\nu+$$
$$+\sum_{\boldsymbol{\tau}=1}^{T}{}'\iint\limits_{\partial\boldsymbol{\Omega}_{b\boldsymbol{\tau}}}\mathbf{n}_b^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)\mathbf{u}_b^\tau(\xi_1^\tau,\xi_2^\tau)d\xi_1^\tau d\xi_2^\tau=$$
$$=\mathbf{t}_b^\nu(x_1^\nu,x_2^\nu)+$$
$$+\sum_{\boldsymbol{\tau}=1}^{T}{}'\iint\limits_{\partial\boldsymbol{\Omega}_{b\boldsymbol{\tau}}}\mathbf{r}_b^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)\mathbf{t}_b^\tau(\xi_1^\tau,\xi_2^\tau)d\xi_1^\tau\xi_2^\tau, \quad (2.8)$$

$$x_1^\nu,x_2^\nu\in\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}};\quad 1\leqslant\nu\leqslant T.$$

$$\iint\limits_{\partial\boldsymbol{\Omega}_{p\boldsymbol{\nu}}}\mathbf{n}_d^\nu(x_1^\nu-\xi_1^\nu,x_2^\nu-\xi_2^\nu)\mathbf{u}_c^\nu(\xi_1^\nu,\xi_2^\nu)d\xi_1^\nu d\xi_2^\nu+$$
$$+\sum_{\boldsymbol{\tau}=1}^{T_1}{}'\iint\limits_{\partial\boldsymbol{\Omega}_{d\boldsymbol{\tau}}}\mathbf{n}_d^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)\mathbf{u}_d^\tau(\xi_1^\tau,\xi_2^\tau)d\xi_1^\tau d\xi_2^\tau+$$
$$+\sum_{\boldsymbol{\tau}=1}^{T_2}{}'\iint\limits_{\partial\boldsymbol{\Omega}_{bd\boldsymbol{\tau}}}\mathbf{n}_d^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)\mathbf{u}_b^\tau(\xi_1^\tau,\xi_2^\tau)d\xi_1^\tau d\xi_2^\tau=$$
$$=\mathbf{t}_c^\nu(x_1^\nu,x_2^\nu)+$$
$$+\sum_{\boldsymbol{\tau}=1}^{T_1}{}'\iint\limits_{\partial\boldsymbol{\Omega}_{d\boldsymbol{\tau}}}\mathbf{r}_d^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)\mathbf{t}_d^\tau(\xi_1^\tau,\xi_2^\tau)d\xi_1^\tau\xi_2^\tau+$$
$$+\sum_{\boldsymbol{\tau}=1}^{T_2}{}'\iint\limits_{\partial\boldsymbol{\Omega}_{bd\boldsymbol{\tau}}}\mathbf{r}_d^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)\,\mathbf{t}_b^\tau(\xi_1^\tau,\xi_2^\tau)d\xi_1^\tau\xi_2^\tau, \quad (2.9)$$

$$x_1^\nu,x_2^\nu\in\partial\boldsymbol{\Omega}_{d\boldsymbol{\nu}};\quad 1\leqslant\nu\leqslant T=T_1+T_2$$

where $c=d$, $p=d$, for x_1^ν, $x_2^\nu\in\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}$;$c=b$, $p=bd$, for x_1^ν, $x_2^\nu\in\partial\boldsymbol{\Omega}_{bd\boldsymbol{\nu}}$

$$\mathbf{N}^{\boldsymbol{\nu}}(\alpha_1^\nu,\alpha_2^\nu)=(\mathbf{D}^{\boldsymbol{\nu}})^{-1}\mathbf{M}^{\boldsymbol{\nu}},$$

$$\mathbf{N}^{\boldsymbol{\nu\tau}}(\alpha_1^\nu,\alpha_2^\nu)=(\mathbf{D}^{\boldsymbol{\nu}})^{-1}\mathbf{M}^\tau,$$
$$\mathbf{R}^{\boldsymbol{\nu\tau}}(\alpha_1^\nu,\alpha_2^\nu)=(\mathbf{D}^{\boldsymbol{\nu}})^{-1}\mathbf{D}^\tau,$$
$$\mathbf{N}^{\boldsymbol{\nu}}(\alpha_1^\nu,\alpha_2^\nu)=(\mathbf{D}^{\boldsymbol{\nu}})^{-1}\mathbf{M}^{\boldsymbol{\nu}},$$
$$\mathbf{N}^{\boldsymbol{\nu\tau}}(\alpha_1^\nu,\alpha_2^\nu)=(\mathbf{D}^{\boldsymbol{\nu}})^{-1}\mathbf{M}^\tau,$$
$$\mathbf{R}^{\boldsymbol{\nu\tau}}(\alpha_1^\nu,\alpha_2^\nu)=(\mathbf{D}^{\boldsymbol{\nu}})^{-1}\mathbf{D}^\tau,$$
$$\mathbf{n}_c^\nu(x_1^\nu,x_2^\nu)=\mathbf{F}_2^{-1}\mathbf{N}_c^\nu(\alpha_1^\nu,\alpha_2^\nu);$$

$$\mathbf{n}_c^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)=\mathbf{F}_2^{-1}\mathbf{N}_c^{\nu\tau}(\alpha_1^\nu,\alpha_2^\nu)\exp i\langle\mathbf{c}_\tau^\nu\boldsymbol{\alpha}^\nu,\boldsymbol{\xi}^\tau\rangle;$$

$$\mathbf{r}_c^{\nu\tau}(x_1^\nu,\xi_1^\tau,x_2^\nu,\xi_2^\tau)=\mathbf{F}_2^{-1}\mathbf{R}_c^{\nu\tau}(\alpha_1^\nu,\alpha_2^\nu)\exp i\langle\mathbf{c}_\tau^\nu\boldsymbol{\alpha}^\nu,\boldsymbol{\xi}^\tau\rangle;$$

$$1\leqslant\nu\leqslant T,\quad c=b,d.$$

An analysis of these formulas shows that the first integral operators on the left are inverted by the integral factorization method presented in [3, 5, 6] and are principal (as in [1, 2]).

Note that, using the above-described derivation of integral equations (2.5), (2.6), and (2.7), (2.8) for a structure consisting of two

blocks, it is not difficult to obtain integral equations for a structure containing an arbitrary number of blocks. Moreover, the system of integral equations for a block structure where domains occupied by blocks are not necessarily convex has a similar form. However, in this case, the principal operators do not need to have kernels dependent on the difference of arguments. In the case of a block structure, as well as in the case of a single body, it is possible to construct an approximate solution discarding small terms. Then, the integral equations can be written as follows:

$$\iint\limits_{\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}} \mathbf{k}_b^\nu(x_1^\nu-\xi_1^\nu, x_2^\nu-\xi_2^\nu)\mathbf{t}_b^\nu(\xi_1^\nu,\xi_2^\nu)d\xi_1^\nu d\xi_2^\nu = \mathbf{u}_b^\nu(x_1^\nu, x_2^\nu) \quad (2.10)$$

$$x_1^\nu, x_2^\nu \in \partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}; \quad 1 \leqslant \nu \leqslant T,$$

$$\iint\limits_{\partial\boldsymbol{\Omega}_{p\boldsymbol{\nu}}} \mathbf{k}_d^\nu(x_1^\nu-\xi_1^\nu, x_2^\nu-\xi_2^\nu)\mathbf{t}_c^\nu(\xi_1^\nu,\xi_2^\nu)d\xi_1^\nu d\xi_2^\nu = \mathbf{u}_c^\nu(x_1^\nu, x_2^\nu)$$

$$x_1^\nu, x_2^\nu \in \partial\boldsymbol{\Omega}_{d\boldsymbol{\nu}}; \quad 1 \leqslant \nu \leqslant T;$$

$$\iint\limits_{\partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}} \mathbf{n}_b^\nu(x_1^\nu-\xi_1^\nu, x_2^\nu-\xi_2^\nu)\mathbf{u}_b^\nu(\xi_1^\nu,\xi_2^\nu)d\xi_1^\nu d\xi_2^\nu = \mathbf{t}_b^\nu(x_1^\nu, x_2^\nu), \quad (2.11)$$

$$x_1^\nu, x_2^\nu \in \partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}; \quad 1 \leqslant \nu \leqslant T$$

$$\iint\limits_{\partial\boldsymbol{\Omega}_{p\boldsymbol{\nu}}} \mathbf{n}_d^\nu(x_1^\nu-\xi_1^\nu, x_2^\nu-\xi_2^\nu)\mathbf{u}_c^\nu(\xi_1^\nu,\xi_2^\nu)d\xi_1^\nu d\xi_2^\nu = \mathbf{t}_c^\nu(x_1^\nu, x_2^\nu),$$

$$x_1^\nu, x_2^\nu \in \partial\boldsymbol{\Omega}_{d\boldsymbol{\nu}}; \quad 1 \leqslant \nu \leqslant T;$$

where $c = d$, $p = d$ for x_1^ν, $x_2^\nu \in \partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}$; and x_1^ν, $x_2^\nu \in \partial\boldsymbol{\Omega}_{b\boldsymbol{\nu}}$; $c = b$, $p = bd$ for x_1^ν, $x_2^\nu \in \partial\boldsymbol{\Omega}_{bd\boldsymbol{\nu}}$.

The modern topological method of solving this problem is presented in [7].

Conclusion

Inverting the integral equations and substituting their accurate or approximate solutions in the integral representations of the solutions to the boundary problems, we have

$$\mathbf{u}_c^\nu = \mathbf{F}_3^{-1}(\mathbf{K}_c(\alpha_1^\nu, \alpha_2^{\boldsymbol{\nu}}, \alpha_3^{\boldsymbol{\nu}}))^{-1} \iint\limits_{\partial\boldsymbol{\Omega}_c} \boldsymbol{\omega}_c^\nu,$$

$$c = b,\ d.$$

Further, one can use the methods described in [1, 2], which make it possible to analyze or calculate two-dimensional integrals.

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