THE THEORY OF THE STARTING EARTHQUAKE

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Abstract. Properties of detected earthquakes which are called starting earthquakes are researched in this work. Questions concerning methods of their detection and their expected behavior, place, time and intensity are discussed. Some characteristic features of real earthquakes are collated with features of model-earthquakes. It is also emphasized that there is no information on this type of earthquakes in scientific literature on hand. Due to the specific nature of the problem, i.e. existence of ruptural and unbounded components in the solution, detection of such earthquakes isn't effective while directly using even competent computing facilities on the solution of a complex of boundary problems in environments with brittle properties. In order to research the earthquakes the application of high-performance computing facilities is suggested when the research is put into topological spaces with a wider range of properties than that of functional spaces.

Keywords: block element, factorization, topology, integral and differential factorization methods, exterior forms, block structures, boundary problems, singularity domain, singular peculiarity, logarithmic singularity, break singularity, algebraic system of equations, earthquake

Introduction

In the work is represented obviously for the first time the model of one type of earthquakes beginning from the preparation to the accomplishment of the event. The model based fully on the laws of physics and mechanics may reveal the new type of faulting earthquake called the starting one, as it precedes to strong crustal earthquakes, connected with the lithosphere plates' interaction. As lithospheric plates we take the Kirchhoff plates on elastic half-space moving to each other till they approach. The earthquake is defined by drastic increase of stress concentration in a specified area in comparison with a normal condition. The mining allows evaluating with the aid of specific equipment the location, time and intensity of this type of earthquakes. The patterns of this earthquake are revealed. The theory of this earthquake is based on the mathematic methods of the high level such as the topology, external analysis, factorization, block element methods. This theory of this earthquake is presented in this paper.

1. Rectangular block elements

The theory of block structures developed at Kuban' State University and the Southern Scientific Center of the Russian Academy of Sciences in [1–4] has a series of advantages discussed below. It allows one to construct the representations of solutions of edge problems for sets of differential equations in partial derivatives in arbitrary regions in analytical form [1–4]. This theory is based on the differential factorization method. For a long time, this method was not noted by scientists developing factorization approaches. A possible cause is that it required modern mathematical methods, particularly, external analysis, the theory of functions of many complex variables, Leray residue forms, factorization of matrix-functions of several complex variables, and the theory of representation of groups. This method is based on fine properties of topological algebra associated with the automorphism of topological varieties, a field of mathematics not often used in applications. In parallel with the differential factorization

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method, the integral factorization method appears [4]. Both methods follow each other in the investigation and solution of both the edge problems and integral equations and their sets [4, 5].

1. The theory of block structures allowed scientists to construct a new method for the investigation and solution of edge problems, which does not repeat previous ones. According to its origin, this method can be called the block element method. It is somewhat similar to the finite element method developed in outstanding works [6, 7], etc. It is noteworthy that a large set of computer programs has been developed on the basis of the finite element method. Among them, for example, ANSYS Mechanical, Multiphysics, Structural, CivilFEM, and AUTODIN should be mentioned. These programs allow one to calculate the solutions of different edge problems of mechanics, physics, ecology, biophysics, engineering applications, and other fields.

However, the finite element method has substantial disadvantages mentioned by its creators themselves [6]. Among the main ones, the replacement of a continuous medium by a finite number of elements of lower dimensionality should be mentioned. As a result, the local description of the solutions of edge problems has only an approximated character. In the finite element method, the carrier should be bounded. The regions of specifying the edge problems also should be bounded. The carriers of the finite element, as a rule, are taken in the two-dimensional case in the form of a triangle or rectangle, including a curvilinear one, and in the three-dimensional case, they are taken in the form of a pyramid, parallelepiped, and possibly curvilinear shape. The functions of the form specified for such a carrier are the polynomials of two or three variables, respectively, which contain several unknown coefficients; i.e., they are splines. The orders of polynomials entering them are dictated by the order of derivatives in differential equations of edge problems. In this case, an important role is played by vertexes, edges, and faces of carriers thus introduced, on which the selected points (nodes) are allocated. They are taken in the forms of sets in the vertexes of the pyramid or triangle, as well as on the edges or faces. The shape functions [6, 7], i.e., the polynomials, are constructed from the requirement of unique determination of coefficients by their values in the carrier nodes. However, several disadvantages of this method should be noted. The polynomials of the spline describe the solution of the edge problem in the carrier region only approximately. This presence of the shape functions of a finite element in the polynomial form does not allow one to analyze the wave components of the solution, especially in edge problems for media with numerous effects on the medium by different physical fields. In addition, if the solution contains strongly oscillating functions, the polynomial shape function cannot represent them correctly. These problems become topical in connection with frequently encountered compositions of microsized and nanosized materials. An increase in the order of derivatives in differential equations complicates the construction of finite elements and their shape functions [6] A decrease in the sizes of splines of finite elements worsens the convergence of the computational processes [8]. However, a great advantage of the finite element method is the banding or almost diagonality of the "rigidity" matrix appearing in this method, which substantially facilitates the computational process. Numerous computational programs mentioned above, undoubtedly, put the finite element method into a number of very effective modern computational means.

The block element is free of the main disadvantage of the finite element – it preserves the medium continuity, which manifests itself exactly satisfying the corresponding differential equations of edge problems [1–4].

Similarly to the finite element, the block element has a carrier, out of which it equals zero. However, its carrier can be any region, namely, bounded, semi-bounded or unbounded, with the boundaries passing into infinity. The carriers of the block element can be both convex regions in the case of exponential factorization and multiply connected regions in the case of the generalized factorization [9]. The block elements are constructed by a definite algorithm of the same type for the sets of differential equations in partial derivatives of any finite order. They have representation in the form of the integral over the boundary of the carrier region [1–4]. Differential equations of the corresponding edge problems can have arbitrary order of derivatives, and they are not associated with the presence of functionals – energy integrals – in them. The application of the finite element method for such edge problems is very complicated.

2. The block element method does not repeat another important computational method, specifically, the boundary element method [10–

12]. The latter implies the construction of the fundamental solution of differential equations, which carries the singular and other features on the boundary of the region of the edge problem under consideration [10–12]. The amount and properties of the features rise, and the order of derivatives increases. The difference between these two methods also consists in satisfying the boundary conditions; it is functional in the boundary element method and topological but leading to the same result in the block element method. In addition, factorization in the block element method remains in the representation the only required components, while the obtained pseudodifferential equations are not only regularized rather simply but are even investigated analytically and admit different variants of approximated solutions [1–4]. In the boundary element methods, it is necessary to solve integral equations with complex, including singular, features, in the case of high-order derivatives in differential equations. The block element method, which is generated by the block structure introduced by a network dividing the region of the solution of the edge problems into the blocks, similarly to the finite element method, leads to the almost diagonal set of pseudodifferential equations. We can also separate other advantages of this method; but as the main one, we can call its investigatory possibilities, which allow one to analyze the solutions of edge problems not restoring to concrete calculations. The method allows one to extend the solution over the whole region under study.

Note that the block element method, similarly to other methods, has some disadvantages. The main disadvantage is that the block element as a function belongs to the space of gradually rising generalized functions $\mathbf{H}_s$ [1–4]. However, this disadvantage can easily be overcome via ignoring the generalized function, whose origin is associated with differentiation of the step function at the boundary of the carrier of the block element. The classical component of the solution continues with the conservation of the required smoothness from block to block.

3. As an example of construction of the block element, let us consider the following two-dimensional edge problem in the restricted region Ω with a smooth boundary $\partial\Omega$ for the differential equation of the form

$$\big[A_{11}(x_1,x_2)\partial^2 x_1 + A_{22}(x_1,x_2)\partial^2 x_2 + A(x_1,x_2)\big]\varphi(x_1,x_2) = 0 \quad (1.1)$$

with certain boundary conditions, for example, Dirichlet or Neumann. Here, the coefficients $A_{kk}(x_1,x_2)$, $A(x_1,x_2)$ are positive smooth functions.

Let us introduce in the region Ω a rectangular network so dense that in the zone of interest of this region, the coefficients $A_{kk}(x_1,x_2)$, $A(x_1,x_2)$ can be assumed constant and denote them A_{kk}, A, while the region of the selected rectangle of the network is Ω_0 with the boundary $\partial\Omega_0$. Let it be described by the relations $|x_1| \leqslant a$ and $|x_2| \leqslant b$ in the initial coordinate system.

In the region Ω_0, let us solve edge problem (1.1) by the differential factorization method applying the algorithm described in [4]. During its application, the tangent fibration of the oriented boundary $\partial\Omega_0$ is performed, and right local coordinates with external normals x_2^k and tangents x_1^k are introduced. The local coordinates are arranged on the rectangle sides and follow counterclockwise with the initial index $k=1$ on the upper side.

Let us introduce the Fourier transform operators

$$\mathbf{F}(\alpha_1^k)\varphi = \int_{-\infty}^{\infty} \varphi(x_1^k)\exp i\alpha_1^k x_1^k \, \mathrm{d}x_1^k,$$

$$\mathbf{F}^{-1}(x_1^k)\varphi = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(\alpha_1^k)\exp(-i\alpha_1^k x_1^k) \, \mathrm{d}\alpha_1^k,$$

$$\mathbf{F}(\alpha_1^k,\alpha_2^k)\varphi = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(x_1^k,x_2^k) \cdot \exp i(\alpha_1^k x_1^k + \alpha_2^k x_2^k)\, \mathrm{d}x_1^k\, \mathrm{d}x_2^k,$$

$$\mathbf{F}^{-1}(x_1^k,x_2^k)\varphi = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(\alpha_1^k,\alpha_2^k) \cdot \exp\Big[-i(\alpha_1^k x_1^k + \alpha_2^k x_2^k)\Big]\, \mathrm{d}\alpha_1^k\, \mathrm{d}\alpha_2^k,$$

$$k = 1,2,3,4.$$

Applying the procedure of construction of automorphism of varieties in the factorization method and calculating the Leray residue forms, let us obtain in local sets of coordinates the pseudodifferential equations of the block element in the form

$$\mathbf{F}^{-1}(x_1^1)\left\{\int\limits_{-a}^{a}\left[-A_{22}\left(\varphi_{12}'-i\alpha_{2-}^1\varphi_1\right)\right.\right.$$
$$\cdot\exp i\alpha_1^1\eta_1^1\,\mathrm{d}\eta_1^1$$
$$-A_{22}\left(\varphi_{32}'+i\alpha_{2-}^1\varphi_3\right)$$
$$\left.\cdot\exp\left[-i\left(2\alpha_{2-}^1b+\alpha_1^1x_1^3\right)\right]\right]\mathrm{d}x_1^3$$
$$-\int\limits_{-b}^{b}\left[A_{11}\left(\varphi_{22}'+i\alpha_1^1\varphi_2\right)\right.$$
$$\left.\cdot\exp\left[-i\left(\alpha_{2-}^1b-\alpha_{2-}^1x_1^2+\alpha_1^1a\right)\right]\right]\mathrm{d}x_1^2$$
$$-A_{11}\left(\varphi_{42}'-i\alpha_1^1\varphi_4\right)$$
$$\cdot\exp\Big[-i\big(\alpha_{2-}^1b$$
$$\left.+\alpha_{2-}^1x_1^4-\alpha_1^1a\big)\Big]\,\mathrm{d}x_1^4\right\}=0, \qquad (1.2)$$
$$x_1^1\in[-a,a],$$

$$\mathbf{F}^{-1}(x_1^2)\left\{\int\limits_{-b}^{b}\left[-A_{11}\left(\varphi_{22}'-i\alpha_{2-}^2\varphi_2\right)\right.\right.$$
$$\cdot\exp i\alpha_1^2\eta_1^2\,\mathrm{d}\eta_1^2$$
$$-A_{11}\left(\varphi_{42}'+i\alpha_{2-}^2\varphi_4\right)$$
$$\left.\cdot\exp i\left(-2\alpha_{2-}^2a-\alpha_1^2x_1^4\right)\,\mathrm{d}x_1^4\right]$$
$$+\int\limits_{-a}^{a}\left[A_{22}\left(-\varphi_{12}'+i\alpha_1^2\varphi_1\right)\right.$$
$$\left.\cdot\exp i\left[-\left(\alpha_{2-}^2a-\alpha_1^2b+\alpha_{2-}^2x_1^1\right)\right]\right]\mathrm{d}x_1^1$$
$$+A_{22}\left(-\varphi_{32}'-i\alpha_1^2\varphi_3\right)$$
$$\cdot\exp i\Big[-\big(\alpha_{2-}^2a$$
$$\left.+\alpha_1^2b-\alpha_{2-}^2x_1^3\big)\Big]\,\mathrm{d}x_1^3\right\}=0, \qquad (1.3)$$
$$x_1^2\in[-b,b],$$

$$\mathbf{F}^{-1}(x_1^3)\left\{\int\limits_{-a}^{a}\left[-A_{22}\left(\varphi_{32}'-i\alpha_{2-}^3\varphi_3\right)\right.\right.$$
$$\cdot\exp i\alpha_1^3\eta_1^3\,\mathrm{d}\eta_1^3$$
$$-A_{22}\left(\varphi_{12}'+i\alpha_{2-}^3\varphi_1\right)$$
$$\left.\cdot\exp i\left[-\left(2\alpha_{2-}^3b+\alpha_1^3x_1^1\right)\right]\right]\mathrm{d}x_1^1$$
$$+\int\limits_{-b}^{b}\left[-A_{11}\left(\varphi_{22}'-i\alpha_1^3\varphi_2\right)\right.$$
$$\left.\cdot\exp i\left[-\left(\alpha_{2-}^3b-\alpha_1^3a+\alpha_{2-}^3x_1^2\right)\right]\right]\mathrm{d}x_1^2$$
$$-A_{11}\left(\varphi_{42}'+i\alpha_1^3\varphi_4\right)$$
$$\cdot\exp i\Big[-\big(\alpha_{2-}^3b$$
$$\left.+\alpha_1^3a-\alpha_{2-}^3x_1^4\big)\Big]\,\mathrm{d}x_1^4\right\}=0, \qquad (1.4)$$
$$x_1^3\in[-a,a],$$

$$\mathbf{F}^{-1}(x_1^4)\left\{\int\limits_{-b}^{b}\left[-A_{11}\left(\varphi_{42}'-i\alpha_{2-}^4\varphi_3\right)\right.\right.$$
$$\cdot\exp i\alpha_1^4\eta_1^4d\eta_1^4$$
$$-A_{11}\left(\varphi_{22}'+i\alpha_{2-}^4\varphi_2\right)$$
$$\left.\cdot\exp i\left[-\left(2\alpha_{2-}^4a+\alpha_1^4x_1^2\right)\right]\right]\mathrm{d}x_1^2$$
$$+\int\limits_{-a}^{a}\left[-A_{22}\left(\varphi_{12}'+i\alpha_1^4\varphi_1\right)\right.$$
$$\left.\cdot\exp i\left[-\left(\alpha_{2-}^4a+\alpha_1^4b-\alpha_{2-}^4x_1^1\right)\right]\right]\mathrm{d}x_1^1$$
$$+A_{22}\left(\varphi_{32}'-i\alpha_3^4\varphi_3\right)$$
$$\cdot\exp i\Big[-\big(\alpha_{2-}^4a$$
$$\left.-\alpha_1^4b+\alpha_{2-}^4x_1^3\big)\Big]\,\mathrm{d}x_1^3\right\}=0, \qquad (1.5)$$
$$x_1^4\in[-b,b].$$

Not going into details, let us note that this class of equations can be investigated and solved by the method of study [5]. The general representation of the solution $\varphi_k(x_1^k,x_2^k)$, i.e., of the block element, after conversion of the set of pseudodifferential equations, can be represented in each local coordinate system x_1^k, x_2^k in the form

$$\varphi_1(x_1^1,x_2^1)=\mathbf{F}^{-1}(x_1^1,x_2^1)K_1^{-1}$$
$$\cdot\left\{\int\limits_{-a}^{a}\Big[-A_{22}\left(\varphi_{12}'-i\alpha_2^1\varphi_1\right)\exp i\alpha_1^1\eta_1^1\,\mathrm{d}\eta_1^1\right.$$
$$-A_{22}\left(\varphi_{32}'+i\alpha_2^1\varphi_3\right)$$
$$\cdot\exp\left[-i\left(2\alpha_2^1b+\alpha_1^1x_1^3\right)\right]\,\mathrm{d}x_1^3\Big]$$
$$-\int\limits_{-b}^{b}\Big[A_{11}\left(\varphi_{22}'+i\alpha_1^1\varphi_2\right)$$
$$\cdot\exp\left[-i\left(\alpha_2^1b-\alpha_2^1x_1^2+\alpha_1^1a\right)\right]\,\mathrm{d}x_1^2$$
$$-A_{11}\left(\varphi_{42}'-i\alpha_1^1\varphi_4\right)$$
$$\left.\cdot\exp\left[-i\left(\alpha_2^1b+\alpha_2^1x_1^4-\alpha_1^1a\right)\right]\,\mathrm{d}x_1^4\Big]\right\},$$

$$\varphi_2(x_1^2, x_2^2) = \mathbf{F}^{-1}(x_1^2, x_2^2)K_2^{-1}$$
$$\cdot \left\{ \int_{-b}^{b} \left[-A_{11}\left(\varphi'_{22} - i\alpha_2^2\varphi_2\right) \exp i\alpha_1^2\eta_1^2 \, \mathrm{d}\eta_1^2 \right.\right.$$
$$- A_{11}\left(\varphi'_{42} + i\alpha_2^2\varphi_4\right)$$
$$\left. \cdot \exp i\left(-2\alpha_2^2 a - \alpha_1^2 x_1^4\right) \, \mathrm{d}x_1^4 \right]$$
$$+ \int_{-a}^{a} \left[A_{22}\left(-\varphi'_{12} + i\alpha_1^2\varphi_1\right) \right.$$
$$\cdot \exp i\left[-\left(\alpha_2^2 a - \alpha_1^2 b + \alpha_2^2 x_1^1\right)\right] \, \mathrm{d}x_1^1$$
$$- A_{22}\left(\varphi'_{32} + i\alpha_1^2\varphi_3\right)$$
$$\left.\left. \cdot \exp i\left[-\left(\alpha_2^2 a + \alpha_1^2 b - \alpha_2^2 x_1^3\right)\right] \, \mathrm{d}x_1^3 \right] \right\},$$
$$\varphi_3(x_1^3, x_2^3) = \mathbf{F}^{-1}(x_1^3, x_2^3)K_1^{-1}$$
$$\cdot \left\{ \int_{-a}^{a} \left[-A_{22}\left(\varphi'_{32} - i\alpha_2^3\varphi_3\right) \exp i\alpha_1^3\eta_1^3 \, \mathrm{d}\eta_1^3 \right.\right.$$
$$- A_{22}\left(\varphi'_{12} + i\alpha_2^3\varphi_1\right)$$
$$\left. \cdot \exp i\left[-\left(2\alpha_2^3 b + \alpha_1^3 x_1^1\right)\right] \, \mathrm{d}x_1^1 \right]$$
$$+ \int_{-b}^{b} \left[-A_{11}\left(\varphi'_{22} - i\alpha_1^3\varphi_2\right) \right.$$
$$\cdot \exp i\left[-\left(\alpha_2^3 b - \alpha_1^3 a + \alpha_2^3 x_1^2\right)\right] \, \mathrm{d}x_1^2$$
$$- A_{11}\left(\varphi'_{42} + i\alpha_1^3\varphi_4\right)$$
$$\left.\left. \cdot \exp i\left[-\left(\alpha_2^3 b + \alpha_1^3 a - \alpha_2^3 x_1^4\right)\right] \, \mathrm{d}x_1^4 \right] \right\},$$
$$\varphi_4(x_1^4, x_2^4) = \mathbf{F}^{-1}(x_1^4, x_2^4)K_2^{-1}$$
$$\cdot \left\{ \int_{-b}^{b} \left[-A_{11}\left(\varphi'_{42} - i\alpha_2^4\varphi_4\right) \exp i\alpha_1^4\eta_1^4 \, \mathrm{d}\eta_1^4 \right.\right.$$
$$- A_{11}\left(\varphi'_{22} + i\alpha_2^4\varphi_2\right)$$
$$\left. \cdot \exp i\left[-\left(2\alpha_2^4 a + \alpha_1^4 x_1^2\right)\right] \, \mathrm{d}x_1^2 \right]$$
$$+ \int_{-a}^{a} \left[-A_{22}\left(\varphi'_{12} + i\alpha_1^4\varphi_1\right) \right.$$
$$\cdot \exp i\left[-\left(\alpha_2^4 a + \alpha_1^4 b - \alpha_2^4 x_1^1\right)\right] \, \mathrm{d}x_1^1$$
$$+ A_{22}\left(\varphi'_{32} - i\alpha_3^4\varphi_3\right)$$
$$\left.\left. \cdot \exp i\left[-\left(\alpha_2^4 a - \alpha_1^4 b + \alpha_2^4 x_1^3\right)\right] \, \mathrm{d}x_1^3 \right] \right\},$$

$$K_1(\alpha_1^m, \alpha_2^m) = A_{11}(\alpha_1^m)^2 + A_{22}(\alpha_2^m)^2 - A,$$
$$m = 1, 3,$$
$$K_2(\alpha_1^n, \alpha_1^n) = A_{22}(\alpha_1^n)^2 + A_{11}(\alpha_2^n)^2 - A,$$
$$n = 2, 4,$$
$$\alpha_{2\pm}^m(\alpha_1^m) = \pm i\sqrt{A_{22}^{-1}(A_{11}(\alpha_1^m)^2 - A)},$$
$$m = 1, 3,$$
$$\alpha_{2\pm}^n(\alpha_1^n) = \pm i\sqrt{A_{11}^{-1}(A_{22}(\alpha_1^n)^2 - A)},$$
$$n = 2, 4.$$

Here, we accept the following notations. In the derivative φ'_{kn} and function φ_k, the first index denotes the number of the coordinate system in which the function is considered and the second one denotes the coordinate along which differentiation is performed. It is clear that when using the formulas of transition from one coordinate system to another one, we obtain the same function $\varphi(x_1, x_2)$. From the integral representation of the solution, it is evident that it is available for analytical investigation in each of introduced local coordinate systems.

2. The Ball Block Elements

We construct block elements with a spherical boundary by the differential factorization method. Contrary to the approaches described in [1–4], where simple factorization related to the representation of the group of translational motions of space was carried out, we applied generalized factorization [5] in this case. This approach is dictated by using the representation of the group of rotations of space induced by the sphere automorphism as a manifold with an edge. As in [1–4], we construct the functional and pseudo-differential equations for describing the block element as well as the representation of the solution for the boundary-value problem. Below, without repeating the general case [5], we presented the block elements for the ball and the space with the cutout ball and the Helmholtz equations derived for the boundary-value problems. The case under consideration is convenient because it makes possible to demonstrate the use of the method for problems solvable by other approaches. When using the method, we open distinctive features of the simple and generalized factorizations the application of the methods in an unlimited region, and the feature of satisfying the boundary conditions. The choice of the equation for the boundary-value problem is related also to the fact that the solutions of precisely this equation are the components of solutions of a number of boundary-value problems of a deformable-solid dynamic.

1. For an illustration, as an example, we constructed here the block elements for the boundary-value problem in the spherical region Ω_1 of radius b and in the space with the cutout spherical region Ω_2 of the radius a with the boundaries $\partial\Omega_s$, $s = 1, 2$, for the Helmholtz differential equation in the form of

$$\mathbf{Q}(\partial x_1, \partial x_2, \partial x_3)\varphi = \left[\partial^2 x_1 + \partial^2 x_2 + \partial^2 x_3 + k^2\right] \cdot \psi(x_1, x_2, x_3) = 0. \quad (2.1)$$

It is shown below that the pseudo-differential equations for the block element enable us to consider all possible variants of boundary conditions θ, φ, r for the partial differential equation. For this purpose, we considered both the Dirichlet and Neumann boundary conditions as in the previous problems.

In the spherical system of coordinates θ, φ, r, Eq.(2.1) for the ball has the form

$$(\Delta + k_1^2)\psi = 0,$$

$$\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\cdot\frac{1}{\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}, \quad (2.2)$$

$$r, \theta, \varphi \in \Omega_1.$$

A similar equation for the half-space with a cavity is taken in the form of

$$(\Delta + k_2^2)w = 0, \quad r, \theta, \varphi \in \Omega_2. \quad (2.3)$$

The solutions of the boundary-value problems for Eqs.(2.2), (2.3) are found in the spaces of slowly increasing generalized functions $\mathbf{H}S$. For investigating this equation by the differential factorization method, we introduce the Fourier–Bessel transform and reversion in spherical functions of the form of

$$\mathbf{B}_2(l, m) = \int_0^{\pi}\int_0^{2\pi} g(\theta, \varphi)Y_l^{m-}(\theta, \varphi)\sin\theta \,\mathrm{d}\theta\,\mathrm{d}\varphi = G(l, m),$$

$$\mathbf{B}_2^{-1}(\theta, \varphi)G = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} G(l, m)Y_l^{m+}(\theta, \varphi) = g(\theta, \varphi),$$

$$\mathbf{B}_3(\lambda, l, m)g = \int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} g(r, \theta, \varphi)J_{l+\frac{1}{2}}(\lambda r) \cdot Y_l^{m-}(\theta, \varphi)\sin\theta \,\mathrm{d}\theta\mathrm{d}\varphi\, r\,\mathrm{d}r = G(\lambda, l, m), \quad (2.4)$$

$$\mathbf{B}_3^{-1}(r, \theta, \varphi)G = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\int_0^{\infty} G(\lambda, l, m)J_{l+\frac{1}{2}}(\lambda r) \cdot Y_l^{m+}(\theta, \varphi)\lambda\,\mathrm{d}\lambda = g(r, \theta, \varphi).$$

Here $J_\nu(\lambda r)$ is the Bessel function, and $Y_l^m(\theta, \psi)$ is the spherical function,

$$Y_l^{m\pm}(\theta, \varphi) = \frac{1}{2}\sqrt{\frac{2l+1}{\pi}\frac{(l-|m|)!}{(l+|m|)!}}P_l^{|m|}(\cos\theta)e^{\pm im\varphi}.$$

Applying transforms (2.3) to Eq.(2.2), we construct the external form [6, 7], which becomes (P, Q, R – some functions)

$$\omega = Pb^2\sin\theta\,\mathrm{d}\theta\wedge\mathrm{d}\varphi + Qb\,\mathrm{d}r\wedge\mathrm{d}\theta + Rb\sin\theta\,\mathrm{d}\varphi\wedge\mathrm{d}r.$$

We carry out the transition to the functional equation. It can be represented in the form [6,7]

$$K(\lambda)\Psi(l, m, \lambda) = \int_{\partial\Omega}\omega, \qquad K(\lambda) = \lambda^2 - k_1^2. \quad (2.5)$$

In the case of a ball, we have

$$(\lambda^2 - k^2)\Psi(l, m, \lambda) = L_{lm}(\lambda),$$

$$L_{lm}(\lambda)b^2\psi'_{lm}(b)T_{lm}(\lambda, b) - b^2\psi_{lm}(b)T'_{lm}(\lambda, b), \quad (2.6)$$

$$\psi_{lm}(r) = \mathbf{B}_2(l,m)\psi(r,\theta,\varphi),$$

$$T_{lm}(\lambda,r) = \frac{1}{\sqrt{r}} J_{l+\frac{1}{2}}(\lambda r).$$

For providing the automorphism and obtaining the pseudo-differential equation, we construct the representation of the boundary-value problem solution as

$$\psi(r,\theta,\varphi) = \mathbf{B}_3^{-1}(r,\theta,\varphi)\frac{L_{lm}(\lambda)}{(\lambda^2 - k_1^2)}. \qquad (2.7)$$

The automorphism requirement consists in fulfilling the equality [6, 7]

$$\psi(r,\theta,\varphi) = 0, \quad r > b. \qquad (2.8)$$

As a result of simple transformations for the simple problem under consideration, we obtain a pseudo-differential equation degenerated into an algebraic one in the form of

$$L_{lm}(k_1) = 0. \qquad (2.9)$$

In complex spatial problems, this equation is pseudo-differential literally.

By the example of this problem, it is already possible to observe the difference of the generalized factorization from the simple one: although the characteristic equation $K(\lambda)$ has two roots, Eq.(2.9) should be fulfilled only for one. A similar problem considered by simple factorization in a layer would require fulfilling Eq.(2.9) for both roots.

Using pseudo-differential Eq.(2.9), we consider the formulation of the boundary-value problems for Eq.(2.2). In the case of setting the Dirichlet conditions for the boundary $\partial\Omega$ for example, in the form of

$$\psi(b,\theta,\varphi) = \psi_0(b,\theta,\varphi). \qquad (2.10)$$

The solution of pseudo-differential Eq.(2.9) is obtained in the form of

$$\psi'_{lm}(b) = \frac{\psi_{lm0}(b)T'_{lm}(k_1,b)}{T_{lm}(k_1,b)}.$$

Here we accept the designation

$$\psi_{lm0}(b) = \mathbf{B}_2(l,m)\psi_0(b,\theta,\varphi).$$

In the complex spatial problems, we obtain the integral or integro-differential equation instead of the algebraic one. For example, we applied the integral factorization method [8] for its solution.

Introducing this relation into Eq.(2.7) and carrying out the necessary calculations, we obtain that, for $r \to b$, there takes place

$$\psi(r,\theta,\varphi) \to \psi_0(b,\theta,\varphi). \qquad (2.11)$$

By using precisely the same algorithm, we solved the problem with the Neumann boundary condition. In this case, instead of boundary condition (2.10), the condition for the derivative is set on the boundary; i.e.,

$$\psi'(b,\theta,\varphi) = \psi_1(b,\theta,\varphi).$$

The solution of the pseudo-differential equation has the form of

$$\psi_{lm}(b) = \frac{\psi'_{lm1}(b)T_{lm}(k_1,b)}{T'_{lm}(k_1,b)}.$$

Here

$$\psi_{lm1}(b) = \mathbf{B}_2(l,m)\psi_1(b,\theta,\varphi).$$

Introducing this relation into Eq.(2.7) and carrying out the transformations, we again obtain the fulfillment of boundary conditions as in the Dirichlet problem; i.e.,

$$\psi'(r,\theta,\varphi) \to \psi_1(b,\theta,\varphi), \quad r \to b, \qquad (2.12)$$

but for the classical component of the solution.

Remark 1. We particularly note that found solution (2.7) of the boundary-value problem in the space of slowly increasing generalized functions $\mathbf{H}_S$ consists of the classical component and the generalized functions. If the initial boundary-value problems are formulated for sufficiently smooth boundary conditions, for example, providing that the solutions belong to the Sobolev's spaces, the classical component coincides with this solution. The generalized component appears only as a result of the differentiation on the normal to the boundary $\partial\Omega$ of the step-function carrier. This circumstance is explained in detail in [9] and also in our subsequent studies.

Relation (2.12) is obtained with taking into account the remark made.

Thus, the pseudo-differential equation involves all possible variants of formulation of the boundary conditions of the problem.

2. We consider the boundary-value problem for the space with a spherical cavity of radius a for Eq.(2.3). Contrary to the finite-element method, the block element can occupy even an unlimited region. The requirement consists in the fact that the region be a manifold with an edge and admits the automorphism. We note that the boundary-value problem under consideration in the unlimited region with material k_2 requires fulfilling the condition of radiation at infinity for correctness of the formulation, for which it is possible to use, for example, the ultimate-absorption principle [10]. This principle dictates the requirement of the corresponding arrangement of the integration contour on the parameter λ in representation (2.4) of the operator $\mathbf{B}_3^{-1}$. The contour should bend around the material pole from below. Further, this property of arranging the integration contour is considered as accepted and taken into account when using the indicated operator.

Similar to the previous case, we construct functional Eq.(2.5) with the same representation of external forms. For this boundary-value problem, it becomes

$$(\lambda^2 - k^2)W(l,m,\lambda) = N_{lm}(\lambda),$$

$$N_{lm}(\lambda) = b^2 w'_{lm}(a)P_{lm}(\lambda,a) - b^2 w_{lm}(a)P'_{lm}(\lambda,a), \quad (2.13)$$

$$w_{lm}(r) = \mathbf{B}_2(l,m)w(r,\theta,\varphi),$$

$$P_{lm}(\lambda,r) = \frac{1}{\sqrt{r}} H^{(1)}_{l+\frac{1}{2}}(\lambda r).$$

Here $H^{(1)}_\nu(\lambda r)$ is the Hankel function of the first kind.

The general representation of the solution can be written as

$$w(r,\theta,\varphi) = \mathbf{B}_3^{-1}(r,\theta,\varphi)\frac{N_{lm}(\lambda)}{(\lambda^2 - k_2^2)}. \quad (2.14)$$

The automorphism of the manifold with an edge (the spaces with the ball removed) is provided if

$$w(r,\theta,\varphi) = 0, \quad r < a.$$

This requirement results in the following pseudo-differential equation:

$$w'_{lm}(a)P_{lm}(\lambda,a) - w_{lm}(a)P'_{lm}(\lambda,a) = 0. \quad (2.15)$$

Similarly to the previous boundary-value problem in the region Ω_2 under consideration, the differential factorization method can be used for solving the boundary-value problems with both Dirichlet and Neumann boundary condition repeating almost everything from the previous section.

In the case of setting the Dirichlet boundary condition in the form of

$$w(a,\theta,\varphi) = w_0(a,\theta,\varphi)$$

the solution of pseudo-differential Eq.(2.15) is represented as

$$w'_{lm}(a) = \frac{w_{lm0}(a)P'_{lm}(k_2,a)}{P_{lm}(k_2,a)}.$$

Here,

$$w_{lm0}(a) = \mathbf{B}_2(l,m)w_0(a,\theta,\varphi).$$

The substitution of the solution into Eq.(2.14) results for $r \to a$ and $r > a$ in the relation

$$w(r,\theta,\varphi) \to w_0(a,\theta,\varphi).$$

As is stated in Remark 1, this convergence takes place under the smooth boundary conditions in the spaces of continuous functions.

When setting the Neumann boundary condition as

$$w'(a,\theta,\varphi) = w_1(a,\theta,\varphi)$$

the solution of the pseudo-differential equation is represented as

$$w_{lm}(a) = \frac{w'_{lm0}(a)P_{lm}(k_2,a)}{P'_{lm}(k_2,a)}.$$

Here,

$$w_{lm1}(a) = \mathbf{B}_2(l,m)w_1(a,\theta,\varphi).$$

According to Remark 1, the introduction of this solution into relation (2.14) results in the following convergence for the classical component of the solution for $r \to a$, $r > a$:

$$w'(r,\theta,\varphi) \to w_1(a,\theta,\varphi).$$

The block elements constructed above can used for the formation of a more complex block structure conjugating the last ones. For example, taking $a > b$ and organizing the contact of surfaces of blocks, we obtain the spherical bearing pair.

It is possible also to construct more complex constructions. The problems of conjugating the block elements are relatively simple and are similar to conjugation in the layered structures; certain examples are presented in [6, 7, 11]. The boundary-value problems for more complex systems of the partial differential equations are considered similarly.

3. The cylindrical block elements

Block elements for the inner and outer boundary problems with the cylindrical boundary are constructed by the differential factorization method. According to the approach described in [1,2], pseudodifferential equations and representation of the solution of the boundary problem are constructed. The determining relations for various block elements with a cylindrical surface are presented. The selection of the boundary problem is associated with the fact that the boundary problems for the sets of differential equations in partial derivatives are investigated using the same approach, but this is more complex technically compared with the considered case. It is shown that the block element method is convenient when investigating the behaviour of the block structures in seismology and no less convenient when describing the quantum-mechanical properties of materials [3,4]. Particularly, the constructed pseudodifferential equations for our block elements with a cylindrical boundary preserve the same properties that were established for the block elements with a spherical boundary. We discuss the possibilities of the block element method by the example of the considered boundary problem.

1. Below, as an illustrative example, we constructed the block elements for the inner boundary problem in a confined cylinder Ω_1 with the radius b and in the space with a cut confined cylinder Ω_2 with the radius a, where we consider the outer boundary problem for the Helmholtz differential equation of the form

$$\mathbf{Q}(\partial x_1, \partial x_2, \partial x_3)\varphi = \left[\partial^2 x_1 + \partial^2 x_2 + \partial^2 x_3 + k^2\right] \cdot \psi(x_1, x_2, x_3) = 0. \quad (3.1)$$

It is shown that pseudodifferential equations of the block element allow us to consider all the possible variants of boundary conditions for the differential equation in partial derivatives. For this purpose, we consider the Dirichlet and Neumann boundary conditions similarly to [1,3–7].

In the cylindrical set of coordinates r, φ, z, Eq.(3.1) for the cylinder has the form

$$(\Delta + k_1^2)\psi = 0,$$
$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}, \quad (3.2)$$
$$r, \varphi, z \in \Omega_1.$$

We take a similar equation for the voided space in the form

$$(\Delta + k_2^2)w = 0, \quad r, \varphi, z \in \Omega_2. \quad (3.3)$$

We seek the solutions of boundary problems for Eqs.(3.2) and (3.3) in the spaces of slowly rising generalized functions $\mathbf{H}_s$. In the case of the outer boundary problem, the radiation condition providing the uniqueness of the solution is fulfilled.

To investigate this equation by the differential factorization method, let us introduce the double and triple transformation and the Fourier–Bessel inversion in the form

$$\mathbf{B}_3(\theta, p, \sigma)u = \int_0^b \int_0^{2\pi} \int_{c_1}^{c_2} u(r, \varphi, z) J_p(\theta r) \cdot \exp\left[i(p\varphi + \sigma z)\right] r \, \mathrm{d}r \, \mathrm{d}\varphi \, \mathrm{d}z = U(\theta, p, \sigma),$$

$$\mathbf{B}_3^{-1}(r, \varphi, z)U = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_0^{\infty} \sum_{p=-\infty}^{\infty} U(\theta, p, \sigma) J_p(\theta r) \cdot \exp\left[-i(p\varphi + \sigma z)\right] \theta \, \mathrm{d}\theta \, \mathrm{d}\sigma = u(r, \varphi, z),$$

$$\mathbf{B}_{21}(p, \sigma)u = \int_0^{2\pi} \int_{c_1}^{c_2} u(\varphi, z) J_p(\theta_0 R) \cdot \exp\left[i(p\varphi + \sigma z)\right] R \, \mathrm{d}\varphi \, \mathrm{d}z = U(p, \sigma), \quad (3.4)$$

$$\mathbf{B}_{21}^{-1}(\varphi, z)U = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{p=-\infty}^{\infty} U(p, \sigma) J_p(\theta_0 R) \cdot \exp\left[-i(p\varphi + \sigma z)\right] \theta_0 \, \mathrm{d}\sigma = u(\varphi, z),$$

$$\mathbf{B}_{23}(\theta, p)u = \int_0^b \int_0^{2\pi} u(r, \varphi) J_p(\theta r) \cdot \exp\left[i(p\varphi + \sigma_0 z_s)\right] r \, \mathrm{d}r \, \mathrm{d}\varphi = U(\theta, p),$$

$$\mathbf{B}_{23}^{-1}(r,\varphi)U = \frac{1}{2\pi}\int_0^\infty \sum_{p=-\infty}^{\infty} U(r,p) J_p(\theta r) \cdot \exp\left[-i(p\varphi + \sigma_0 z_s)\right]\theta\,\mathrm{d}\theta = u(r,\varphi).$$

Here, $J_\nu(\lambda r)$ is the Bessel function.

Applying transformations (3.4) to Eq.(3.2), let us construct the exterior form [1,3–6], which looks like

$$\omega = g\left[r\left(\frac{\partial\psi}{\partial r} - \theta J_p'(\theta b)J_p^{-1}(\theta b)\psi\right)\mathrm{d}\varphi\wedge\mathrm{d}z - \frac{1}{r}\left(\frac{\partial\psi}{\partial\varphi} - ip\psi\right)\mathrm{d}r\wedge\mathrm{d}z + r\left(\frac{\partial\psi}{\partial z} - i\sigma\psi\right)\mathrm{d}r\wedge\mathrm{d}\varphi\right], \quad (3.5)$$

$$g(r,\varphi,z) = J_p(\theta r)\exp\left[i(p\varphi + \sigma z)\right]$$

Let us perform the transition to the functional equation and find it in the form

$$K(\theta,\sigma)\Psi(\theta,p,\sigma) = \int_{\partial\Omega_1}\omega, \qquad K(\theta,\sigma) = (\theta^2 + \sigma^2 - k_1^2). \quad (3.6)$$

Let us represent the region boundary in the form $\partial\Omega_1 = \partial\Omega_{10}\cup\partial\Omega_{20}$, where $\partial\Omega_{10}$ is the side cylindrical part of the boundary $\partial\Omega_{10}$, and $\partial\Omega_{20}$ is the end part consisting of two parts, namely, $\partial\Omega_{21}$is the end $z = c_1$ and $\partial\Omega_{22}$ is the end $z = c_2$. Let us introduce the solutions at the region boundaries denoting the values of the function ψ at the boundaries $\partial\Omega_{10}$, $\partial\Omega_{21}$ and $\partial\Omega_{22}$ as ψ_r, ψ_{z1}, ψ_{z2} respectively.

Applying the conventional algorithm of construction of pseudodifferential equations associated with the construction of the tangential separation of the boundary [1,3–7], let us introduce the curvilinear local coordinate systems at each boundary $\partial\Omega_\nu$. The automorphism requirement [2] leads to the following pseudodifferential equations:

$$\mathbf{F}_2^{-1}(r)P_1(\theta,\sigma_-,b,c_1,c_2)\exp\left[-i\sigma_- c_1\right] = 0,$$

$$r\in[0,b], \quad \sigma_\pm = \pm i\sqrt{\theta^2 - k_1^2},$$

$$\mathbf{F}_2^{-1}(r)P_1(\theta,\sigma_+,b,c_1,c_2)\exp\left[-i\sigma_+ c_2\right] = 0$$

$$r\in[0,b],$$

$$P_1(\theta,\sigma,b,c_1,c_2) = \int_{c_1}^{c_2}\left[J_p(\theta b)\frac{\partial\Psi_{rp}}{\partial r} - \theta J_p'(\theta b)\Psi_{rp}\right] b\exp i\sigma z\,\mathrm{d}z + \int_0^b J_p(\theta r)\left[\frac{\partial\Psi_{1p}}{\partial z} - i\sigma\Psi_{1p}\right] r\exp i\sigma c_1\,\mathrm{d}r + \int_0^b J_p(\theta r)\left[\frac{\partial\Psi_{2p}}{\partial z} - i\sigma\Psi_{2p}\right] r\exp i\sigma c_2\,\mathrm{d}r,$$

$$\Psi_{\nu p} = \mathbf{B}_1(r,p,z)\psi_\nu.$$

The derived pseudodifferential equations allow us to form the integral equations for all the possible variants of boundary problems, which can be constructed for differential equation (3.1). This is described in more detail in [8–10]. The solution of these integral or integro-differential equations gives the solution of pseudodifferential equations. Representation of the solution obtained after the inversion of pseudodifferential equations is given by relationship

$$\psi(r,\varphi,z) = \mathbf{B}_3^{-1}(r,\varphi,z)K^{-1}(\theta,\sigma)\int_{\partial\Omega_1}\omega.$$

2. Let us consider the case of the boundary problem for the equation

$$(\Delta + k_2^2)w = 0$$

in the region

$$\Omega_2: \quad a\le r\le\infty,\ -\infty\le z\le c_1,\ c_2\le z\le\infty.$$

In contrast with the case considered in [1] for the outer region having a single-type boundary, the region is outer in this case, while the diverse boundary contains cylindrical and planar components. For this reason, we are forced to use three block elements for the description of the mentioned block structure. It is noteworthy that the block elements can be selected by various methods. The problem is assuring that their number would be minimal and they would describe our zones of interest of the block structure under study in the most favorable manner.

Let us introduce the block elements by two methods. The first is obtained by the dissection of the region Ω_2 by the planes infinitely continuing the cylinder ends. To realize the second case, the cylindrical boundary is infinitely continued in both directions. In the first case, we acquire three block elements in the form of a layer with a cylindrical orifice and two half-spaces. In the second case, we have block elements in the space with a cut cylinder and two semi-infinite cylinders.

Let us introduce the following notation for the solutions at the boundary of the region Ω_2:

$$w(a,\varphi,z)=w_r,\quad c_1\leqslant z\leqslant c_2,$$

$$w(r,\varphi,c_1)=w_1 \text{ and } w(r,\varphi,c_2)=w_2,$$
$$0\leqslant r\leqslant a.$$

Retaining the above-described notation of the boundaries of the cylindrical and repeating the construction of the pseudodifferential equation, we come to their following representation in the first case:

$$\mathbf{F}_1^{-1}(z)P_2(\theta_1,\sigma,b,c_1,c_2)=0,$$

$$z\in[c_1,c_2],\quad \theta_1=i\sqrt{\sigma^2-k_2^2},$$

$$\mathbf{F}_2^{-1}(r)P_2(\theta,\sigma_-,a,c_1,c_2)\exp\left[-i\sigma_-c_1\right]=0,$$
$$r\in[a,\infty],$$
$$\sigma_\pm=\pm i\sqrt{\theta^2-k_2^2},$$
$$\mathbf{F}_2^{-1}(r)P_2(\theta,\sigma_+,a,c_1,c_2)\exp\left[-i\sigma_+c_2\right]=0,$$
$$r\in[a,\infty],$$
$$\mathbf{F}_2^{-1}(r)P_2^-(\theta,\sigma_-,b,c_1,c_2)\exp\left[-i\sigma_-c_1\right]=0,$$
$$r\in[0,\infty],$$
$$\mathbf{F}_2^{-1}(r)P_2^+(\theta,\sigma_+,b,c_1,c_2)\exp\left[-i\sigma_+c_2\right]=0,$$
$$r\in[0,\infty].$$

We here accepted the notation

$$P_2(\theta,\sigma,a,c_1,c_2)$$
$$=\int_{c_1}^{c_2}\left[H_p^{(1)}(\theta a)\frac{\partial W_{rp}}{\partial r}-\theta\{H_p^{(1)}(\theta a)\}'W_{rp}\right]$$
$$\cdot a\exp i\sigma z\,\mathrm{d}z$$
$$+\int_a^\infty J_p(\theta r)\left[\frac{\partial W_{1p}^+}{\partial z}-i\sigma W_{1p}^+\right]r\exp i\sigma c_1\,\mathrm{d}r$$
$$+\int_a^\infty J_p(\theta r)\left[\frac{\partial W_{2p}^+}{\partial z}-i\sigma W_{2p}^+\right]r\exp i\sigma c_2\,\mathrm{d}r,$$

$$W_{\nu p}=\mathbf{B}_1(r,p,z)w_\nu$$

$$P_2^-(\theta,\sigma,a,c_1)$$
$$=\int_0^\infty J_p(\theta r)\left[\frac{\partial(W_{1p}+W_{1p}^+)}{\partial z}-i\sigma(W_{1p}+W_{1p}^+)\right]$$
$$\cdot r\exp i\sigma c_1\,\mathrm{d}r,$$

$$P_2^+(\theta,\sigma,a,c_2)$$
$$=\int_0^\infty J_p(\theta r)\left[\frac{\partial(W_{2p}+W_{2p}^+)}{\partial z}-i\sigma(W_{2p}+W_{2p}^+)\right]$$
$$\cdot r\exp i\sigma c_2\,\mathrm{d}r,$$

$$W_{\nu p}^+=0,\quad r\leqslant a,\quad \nu=1,2.$$

In the second case, we derive the set of pseudodifferential equations

$$\mathbf{F}_1^{-1}(z)P_{21}(\theta_2,\sigma,a)=0,$$
$$z\in[-\infty,\infty],\quad \theta_2=i\sqrt{\sigma^2-k_2^2},$$
$$\mathbf{F}_1^{-1}(z)P_{11}(\theta,\sigma_-,a,c_1)\exp\left[-i\sigma_-c_1\right]=0,$$
$$-\infty\leqslant z\leqslant c_1,$$
$$\mathbf{F}_1^{-1}(z)P_{12}(\theta,\sigma_+,a,c_2)\exp\left[-i\sigma_+c_2\right]=0,$$
$$c_2\leqslant z\leqslant\infty,$$
$$\sigma_\pm=\pm i\sqrt{\theta^2-k_2^2},$$
$$\mathbf{F}_2^{-1}(r)P_{11}(\theta,\sigma_-,a,c_1)\exp\left[-i\sigma_-c_1\right]=0,$$
$$0\leqslant r\leqslant a,$$
$$\mathbf{F}_2^{-1}(r)P_{12}(\theta,\sigma_+,a,c_2)\exp\left[-i\sigma_+c_2\right]=0,$$
$$0\leqslant r\leqslant a,$$

$$P_{21}(\theta,\sigma,a)$$
$$=\int_{-\infty}^\infty\left[H_p^{(1)}(\theta a)\frac{\partial(W_{rp}+W_{rp}^-+W_{rp}^+)}{\partial r}\right.$$
$$\left.-\theta\{H_p^{(1)}(\theta a)\}'(W_{rp}+W_{rp}^-+W_{rp}^+)\right]$$
$$\cdot a\exp i\sigma z\,\mathrm{d}z,$$

$$P_{11}(\theta,\sigma,b,c_1,c_2)$$
$$=\int_{-\infty}^{c_1}\left[J_p(\theta b)\frac{\partial W_{rp}^-}{\partial r}-\theta J_p'(\theta b)W_{rp}^-\right]$$
$$\cdot a\exp i\sigma z\,\mathrm{d}z$$
$$+\int_0^a J_p(\theta r)\left[\frac{\partial W_{1p}}{\partial z}-i\sigma W_{1p}\right]r\exp i\sigma c_1\,\mathrm{d}r,$$

$$P_{12}(\theta,\sigma,b,c_1,c_2) = \int\limits_{c_2}^{\infty}\left[J_p(\theta b)\frac{\partial W_{rp}^{+}}{\partial r} - \theta J_p'(\theta b)W_{rp}^{+}\right] \cdot a\exp i\sigma z\,\mathrm{d}z + \int\limits_{0}^{a} J_p(\theta r)\left[\frac{\partial W_{2p}}{\partial z} - i\sigma W_{2p}\right] r\exp i\sigma c_2\,\mathrm{d}r.$$

$$w_r^{\pm}(a,\varphi,z) = \mathbf{B}_1^{-1}(a,\varphi,z)W_{rp}^{\pm}.$$

Here, functions $w_r^{\pm}(a,\varphi,z)$ differ from zero at $-\infty \leqslant z \leqslant c_1$ in the case of a minus at the superscript and $c_2 \leqslant z \leqslant \infty$ in the case of a plus, respectively. The functions W_{rp}^{-}, W_{rp}^{+}, W_{1p}^{+} participating in these representations are auxiliary and can easily be eliminated from the presented sets of pseudodifferential equations using operators (3.4). However, this should be made after the statement of the problem when the set of boundary conditions is determined. More concretely, it is determined for which fragments of the boundary regions the values of functions, their normal derivatives, or mixed conditions are specified. After this, the pseudodifferential equations are reduced to integral or integro-differential equations. In view of simplicity, this part of transformations is omitted. The representation of the solution is given by the relationship

$$\psi(r,\varphi,z) = \mathbf{B}_3^{-1}(r,\varphi,z)K^{-1}(\theta,\sigma)\int\limits_{\partial\Omega_2}\omega.$$

It is noteworthy that when using the operator $\mathbf{B}_3^{-1}$, the integration contours should be arranged correctly to provide for the radiation conditions at infinity, for example, applying the limiting absorption principle [10, 11].

Thus, similarly to other boundary problems [1, 3–9], pseudodifferential equations serve in the formulation of all the possible types of boundary problems admissible by the differential equation under consideration.

Note 1. The application of the block element method described in our article is given as an illustration, which simultaneously shows its distinction from the finite element method and the boundary element method. The described constructions for the set of differential equations in partial derivatives, which nevertheless require the involvement of factorization of matrix functions, are performed similarly [8, 9].

Note 2. The above-constructed block elements for the block structure, which describe its acoustical properties, allow us to construct by analogy the representation of solutions in any complex acoustic block structure with the totalities of the sources in the complex-shaped regions and to investigate the questions of resonances including addresses. For this purpose, a pre-existing rather large set of block elements should be used [1, 3–7]. In particular, we can state the problems on the directed acoustic antennas [10] in block structures.

Let us note the main advantages of the use of the block element method to solve such problems. These are the possibility to “cut out” the block structure by sections into the block elements depending on the stated problem, the use of a rich apparatus of investigation and solution of integral equations of the mixed problems, and the possibility to represent the solutions in each block in the form of the Fourier integral with the dispersion equation in the denominator of the inegrand, which allows us to exactly describe the wave process in the block structure [10, 11].

4. Automorphism in factorization

When using factorization methods for investigating and solving boundary-value problems in block structures, pseudo-differential equations play an important role. They play the role [1–5] of certain “managers” in formulation and implementation of the boundary-value problem. By varying its parameters, it is possible to set the boundary-value problems, both basic and mixed. Interesting properties of these operators were found when investigating the boundary-value problems of quantum mechanics.

In unsteady boundary-value problems, the pseudo-differential operators transfer the initial conditions into the category of boundary conditions. The pseudo-differential equations in investigating the resonance properties of block structures in the block-element method and in solving boundary-value problems with separating variables by this method.

The method of constructing pseudo-differential equations is based on the requirement of the automorphism of the manifolds with an edge, i.e., the carriers on which the boundary-value problem is formulated. The basic theorem, the proof of which was for the first time published in, is the basis for fulfilling the automorphism. It is of interest to clarify the relation between the automorphism and the pseudo-differential equations using the various methods of their construction. Below by the example of the scalar boundary-value problem in the convex region with a piece-wise smooth boundary for the elliptic partial differential equation with constant coefficients, we construct a pseudo-differential equation of the boundary-value problem by the Wiener–Hopf method and study its relation to the automorphism. It is proved that implementing the automorphism and vanishing the pseudo-differential equation are equivalent requirements.

1. The above boundary problem the initial Cartesian system of coordinates is given by the equation

$$Q(\partial x_1, \partial x_2, \partial x_3)\varphi = \sum_{m=1}^{2M}\sum_{p=1}^{2P}\sum_{n=1}^{2N} A_{mpn}\varphi^{(m)(n)(n)}_{x_1\ x_2\ x_3} = 0, \quad (4.1)$$

$$A_{mpn} = \mathrm{const}, \quad \varphi(\mathbf{x}) = \varphi(x_1, x_2, x_3), \quad \mathbf{x} = \{x_1, x_2, x_3\}, \quad \mathbf{x} \in \Omega.$$

Without repeating the traditional procedure of reducing the boundary-value problem to the functional equation, we write it in the form.

$$K^\nu(\alpha^\nu)\varphi^\nu(\alpha^\nu) = \iint\limits_{\partial\Omega} \boldsymbol{\omega}^\nu,$$

$$-Q(-i\alpha_1^\nu, -i\alpha_2^\nu, -i\alpha_3^\nu) \equiv K^\nu(\alpha_1^\nu, \alpha_2^\nu, \alpha_3^\nu). \quad (4.2)$$

$\boldsymbol{\omega}^\nu$ is the external form generated by the differential equation on the manifold Ω. It is dependent on all derivatives of the function φ up to the order $2N-1$ inclusively considered on the boundary $\partial\Omega$. It is the polynomial $K^\nu(\alpha^\nu) = K^\nu(\alpha_1^\nu, \alpha_2^\nu, \alpha_3^\nu)$ of three complex variables in the system of coordinates x^ν of the boundary tangential stratifications. The $\alpha_1^\nu, \alpha_2^\nu, \alpha_3^\nu$ are the parameters of the Fourier transform $\varphi^\nu(\alpha^\nu)$ of the desired function of the boundary-value problem. The axis x_3^ν is directed along the normal to the boundary.

We designate the zeros of the polynomial lying in the upper half-plane for the plus sign and in the lower half-plane for the minus sign assuming a multiplicity of one. We consider that the number of zeros of each group is N in each system of coordinates. We also consider that a certain portion of the boundary $\partial\Omega$ lies in the plane $x_3^\nu = 0$. The addition of this portion up to the entire boundary is designated as $\partial\Omega_1$.

According to the differential factorization method, (4.2) should be fulfilled in each system of coordinates $\mathbf{x}^\nu$ of the boundary tangential stratifications. Taking into account that Ω is always located in the region $x_3^\nu \leqslant 0$, (4.2) can be rewritten as

$$K^\nu(\alpha^\nu)\varphi_-^\nu(\alpha^\nu) = F_-^\nu, \quad F_-^\nu = \iint\limits_{\partial\Omega} \omega^\nu. \quad (4.3)$$

Here we use the designation of functions regular in the upper (the plus again) or lower (the minus sign) half-planes, which is traditional for the factorization methods [7, 8]. The obtained relation is an incomplete functional Wiener–Hopf equation because there is no second unknown function regular in the upper half-plane.

We apply the technique of solving the functional Wiener–Hopf [7, 8] equations to it. The solution is sought in the class $\mathbf{H}_s$ of slowly increasing generalized functions. We implement the differential factorization of the coefficient $K^\nu(\alpha^\nu)$ of characteristic (4.3) as the function of the parameter α_3^ν.

$$K^\nu(\alpha^\nu) = K_+^\nu(\alpha^\nu)K_-^\nu(\alpha^\nu),$$

$$K_\pm^\nu(\alpha^\nu) = \prod_{r=1}^{N}(\alpha_3^\nu - \alpha_{3r\mp}^\nu).$$

We divide Eq.(4.3) and introduce the designations.

$$\left[K_+^\nu(\alpha^\nu)\right]^{-1} F_{-1}^\nu = \left[K_+^\nu(\alpha^\nu)\right]^{-1} \iint\limits_{\partial\Omega_1} \boldsymbol{\omega}^\nu = D^\nu + T^\nu,$$

$$\left[K_+^\nu(\alpha^\nu)\right]^{-1} F_{-0}^\nu = \left[K_+^\nu(\alpha^\nu)\right]^{-1} \iint\limits_{\partial\Omega_0} \boldsymbol{\omega}^\nu = L^\nu + B^\nu,$$

$$F_-^\nu = F_{-0}^\nu + F_{-1}^\nu.$$

Here, D^ν is the polynomial of the variable α_3^ν of the order $N-1$ with the coefficients in the form of exponential functions dependent on α_3^ν and decreasing in the lower half-plane, T^ν is the remainder of division of the functions F_{-1}^ν, on the $K_+^\nu(\alpha^\nu)$. L^ν is the polynomial with the coefficients dependent on α_1^ν, α_2^ν, and B^ν is a rational function – the remainder of division of the polynomials F^{ν}_{-0} on the $K^{\nu}_{+}(\alpha^{\nu})$.

Applying the conventional factorization techniques, we come to the relations [7,8].

$$\varphi^{\nu}_{-}(\alpha^{\nu})\left[K^{\nu}_{-}(\alpha^{\nu})\right]^{-1} \cdot\left[D^{\nu}+L^{\nu}+T^{\nu}_{-}+B^{\nu}_{-}+G\right], \quad (4.4)$$

$$T^{\nu}_{+}+B^{\nu}_{+}-G=0. \quad (4.5)$$

When constructing formulas (4.4), (4.5), we used the factorization relations in the form of the sum of analytical functions set in a certain regularity band containing the material axis $\operatorname{Im}\alpha^{\nu}_{3}=0$ and having the form [7,8].

$$\left[K^{\nu}_{+}(\alpha^{\nu})\right]^{-1}T^{\nu}=T^{\nu}_{+}+T^{\nu}_{-},$$

$$\left[K^{\nu}_{+}(\alpha^{\nu})\right]^{-1}B^{\nu}=B^{\nu}_{+}+B^{\nu}_{-}.$$

Here G is the polynomial of the order $N-1$ of the variable α^{ν}_{3} with arbitrary coefficients arising due to the estimate of the whole function in the entire plane (Liuvill's Theorem). The coefficients depend on the parameters α^{ν}_{1}, α^{ν}_{2}.

2. We carry out the following relation the pseudo-differential equation of a boundary-value problem in the block-element method.

Definition 1. We call the following relation the pseudo-differential equation of a boundary-value problem in the block-element method.

$$T^{\nu}_{+}+B^{\nu}_{+}. \quad (4.6)$$

Definition 2. The following relation is called the manifold automorphism of a boundary-value problem in the block-element method

$$\begin{gathered}\varphi(\mathbf{x})=\varphi(x_1,x_2,x_3)=0,\\ \mathbf{x}=\{x_1,x_2,x_3\}\notin\Omega.\end{gathered} \quad (4.7)$$

Theorem 1. In the block-element method for boundary-value problem (4.1), the conditions of fulfilling automorphism and vanishing the pseudo-differential equation are equivalent.

Proof. Let us present several methods of proving this theorem.

1) We first consider the region Ω different from the half-space. Then it follows from Eq.(4.2) that the fulfillment of condition (4.4) is possible only under the condition that $G=0$. From here Eq. follows (4.6).

Conversely, if Eq.(4.6) takes place, the equality $G=0$ follows from Eq.(4.5), which provides the automorphism.

Let us consider now the case when the region Ω is a half-plane. In this case, (4.4), (4.5) become

$$\varphi^{\nu}_{-}(\alpha^{\nu})=\left[K^{\nu}_{-}(\alpha^{\nu})\right]^{-1}\left[L^{\nu}+B^{\nu}_{-}+G\right], \quad (4.8)$$

$$B^{\nu}_{+}-G=0. \quad (4.9)$$

Relation (4.8) provides the automorphism because the function $\left[K^{\nu}_{-}(\alpha^{\nu})\right]^{-1}G$ is the Fourier transform of the function with the carrier in the region Ω. Let us consider (4.9). Being the function of the parameter α^{ν}_{3}, it represents the sum of the rational function and the polynomial. The last term can be zero for all α^{ν}_{3} if and only if the following equalities take place,

$$B^{\nu}_{+}=0, \quad G=0.$$

For proving the opposite statement, we rule out the polynomial G from (4.8), (4.9) and obtain the expression in the form of

$$\varphi^{\nu}_{-}(\alpha^{\nu})=\left[K^{\nu}_{-}(\alpha^{\nu})\right]^{-1}\left[L^{\nu}+B^{\nu}_{-}+B^{\nu}_{+}\right].$$

From the last formula, it can be seen that the function $\varphi(\mathbf{x})$ has the carrier in the region Ω if and only if the pseudo-differential equation $B^{\nu}_{+}=0$ vanishes.

2) The second proof is based on the representation of functional (4.3) in the form

$$\varphi^{\nu}_{-}(\alpha^{\nu})=\left[K^{\nu}(\alpha^{\nu})\right]^{-1}F^{\nu}_{-}, \quad (4.10)$$

and the subsequent factorization in the form of the sum of the right-hand side, which gives the relations.

$$\begin{gathered}\left[K^{\nu}(\alpha^{\nu})\right]^{-1}F^{\nu}_{-}=M^{\nu}_{+}+M^{\nu}_{-},\\ \varphi^{\nu}_{-}(\alpha^{\nu})=M^{\nu}_{-}, \quad M^{\nu}_{+}=0.\end{gathered} \quad (4.11)$$

The proof of the theorem is obvious.

3) The third method assumes the direct calculation of the representation of the solution outside of the region Ω in the local system of coordinates and the use of the requirement of its vanishing, i.e., fulfilling automorphism. As a result of transformations, the pseudo-differential equations are obtained from this condition [1,9].

All three methods lead to the same results. □

The proved theorem is generalized to the case of the sets of partial differential equations. In the general vector case, the following is valid.

Theorem 2. In the block-element method for the set of partial linear differential equations with constant coefficients, the conditions of the fulfilling automorphism and vanishing the pseudo-differential equation are equivalent.

The proof of the theorem is similar to those presented above; however, it has specificity. We restrict ourselves to certain marks.

Seemingly the most accessible for constructing the pseudo-differential equation, the second method giving the matrix representation of pseudo-differential (4.11), in the vector case, in practice, gives the degenerated matrix M_+^ν, and the extraction of independent relations from it is quite a challenge [10].

The first method is preferable in the combination with the factorization of the polynomial matrix functions developed in [11,12]. The third method is most general because it can be used also in the curvilinear coordinates instead of only in the Cartesian coordinates. In this case, it is combined with the factorization of the matrix functions. As is known, the class of the boundary-value problems admitting the reduction to the Wiener–Hopf equations is quite restricted. In particular, it is great rarity in curvilinear coordinates. Therefore, there is no hope for obtaining functional equations similar to (4.3). It is preferable to construct the pseudo-differential equations considering the manifold automorphism as more accessible for analysis, especially when the case in point is the boundary-value problems for the sets of partial differential equations, which are difficult in formulation. However, as was stated above, the use of these theorems is justified only in connection with the proof of the theorem about the boundary properties of the solutions of problems under investigation established in [6].

5. General properties of block elements

The results of determination of block elements proposed in [1–4] mainly for boundary-value problems with constant coefficients give the methods of their construction, from which it is possible to see the close relation of these elements to a particular boundary-level problem. This circumstance induces the problem on a possible restriction of using block elements that have their origin in particular boundary-value problems.

In this paper, we present results describing the relation between the block elements for different boundary-value problems, which show that important relations can exist between the block elements of these boundary-value problems. The listed set of properties of block elements enables us to use them more widely in various fields.

1. The possibilities of the block-element method are displayed by its use in a number of polytypic problems presented below.

In [1–4], the concept of a block element is introduced, and a number of examples of particular block elements are given for certain boundary-value problems. It is shown that the block elements are determined by the boundary-value problem and can always be constructed for an unambiguously solvable boundary-value problem formulated for a set of partial differential equations of a finite order with constant coefficients in the region with a piece-smooth boundary [5–7]. They also can be constructed for the boundary-value problems with variable coefficients admitting the separation of variables [8].

In the general case of the boundary-value problems with variable coefficients, their region of formulation of the boundary-value problem is divided by a mesh for using the block-element method. The mesh should be so dense that it could be possible to consider the coefficients in a division cell as constant [1–4, 9].

A certain practice of applying the block elements shows that their use simplifies the formulation of a number of boundary-value problems and also the construction of their solutions. For example, the block-element method makes it possible to solve the boundary-value problems for homogeneous and inhomogeneous sets of partial differential equations in a similar way [7]. For its use, it is unnecessary to construct individually the general solutions of homogeneous differential equations and the partial solutions of inhomogeneous equations with the subsequent fulfillment of boundary conditions. In the unsteady boundary-value problems, the block-element method raises both the edge boundary conditions for sets of partial differential equations and the initial conditions of a boundary-value problem [10] to the rank of boundary conditions; i.e., the initial conditions in the block-element method become the boundary conditions. The block-element method makes it possible to consider the same boundary-value problems in the bounded, semi-bounded, and unbounded regions.

The block elements enable us to simplify the derivation of certain important characteristics of the solution.

For example, the block elements describe the state function and the wave function of an elementary particle in the problems of quantum mechanics [11]. The normalized square of the modulus of its Fourier transform, which requires no calculation, gives the probability of keeping a particle in the block-element-carrier zone. Varying the shape of the block-element carrier, it is possible to obtain quantum-mechanical objects, which are more complicated than the quantum wells, wires and dots [11]. The pseudo-differential equations arising in these problems involve all cases of the particle energy state in the same way.

In the problems of continuum mechanics, the functions on the boundaries of the block-element carrier, which either require a determination or are set and included in the pseudo-differential equations, are the particular physical characteristics of the solution of the boundary-value problem under consideration.

For example, in the problems of elasticity theory, these are the displacements or stresses on the block-element boundary; in the problems of the theory of plates, they are the displacements, angles of rotation, or shear and normal forces and moments. In the boundary-value problems of electrodynamics, it is the electric potential, the electric charge, the tangential component of the electric-field vector, and the normal component of the electric induction.

2. By the example of a particular boundary-value problem, we present certain general properties of the block elements revealing their features and admitting the generalization on the general case. Here and below, considering the construction of solutions of a boundary-value problem by the block-element method, we mean that the solution of the corresponding pseudo-differential equations was constructed.

Let an unambiguously solvable boundary-value problem for the set of the partial differential equations of a finite order with constant coefficients be considered in the convex singly connected polyhedral region Ω with the boundary $\partial\Omega$ [7]. The block elements of such a boundary-value problem represent the vectors, the components of which are block elements similar to the scalar ones in the case of the boundary-value problem for a single differential equation. Further, we do not distinguish these two concepts calling them block elements in both cases.

Let us consider various divisions of the region Ω by the mesh, the boundaries of which represent various planes. As a result, the region Ω is divided into n polyhedral convex regions $\Omega_k(n)$, $k = 1, 2, \dots, n$. Rejecting certain boundaries in the division mesh, we obtain a new division of the region Ω containing a smaller number of larger regions $\Omega_k(p)$, $k = 1, 2, \dots, p$, $p < n$, each of which can be a combination of several regions $\Omega_k(n)$). Continuing the process of elimination of boundaries of the division mesh, we obtain the sequence of divisions $1 < p_1 < p_2 < \dots < n$.

In the case of $p = 1$, we obtain a single cell, which proves to the region Ω.

The number n can be either finite or tend to infinity.

For each division p_r, we designate the block element corresponding to it as $B_k(p_r)$. We introduce the concept of the combination

$$B_k(p_r) = B_l(p_s) \cup B_h(p_s), \quad r < s, \qquad (5.1)$$

for the block elements contacting over the general boundary, which consists in constructing the block element located on the combination of their carriers. The number of united elements can be arbitrarily finite.

After constructing the solutions φ of the boundary-value problem for each of the divisions by the block-element method, for doing which we convert the corresponding pseudo-differential equations [7], we obtain the representation of solutions for each r in one of the following forms:

$$\varphi = \sum_k B_k(p_r), \qquad r = 1, 2, \dots, n, \quad p_1 = 1, \quad p_n = n. \qquad (5.2)$$

The cited formula displays the completeness of the block elements in $\mathbf{H}_s$ for each of the divisions. We consider in more detail (5.1), we obtain the representation of the function φ, which is invariable in the left-hand side and expanded in terms of larger block elements.

Thus, for constructing the solution of the boundary-value problem by the block-element method, it is possible to diversify the choice of divisions of the region Ω in which the boundary-value problem is formulated by the mesh on the basis of the reasons of an optimum selection of the corresponding block elements and their conjugation by means of solving the pseudo-differential equations. The fulfillment of this requirement substantially depends on the shape of the region of formulation of the boundary-value problem and the type of the differential equations for which it is formulated.

The practice of application of the block-element method shows that for constructing the block elements, it is possible in certain cases also to use other methods, which enable us to implement more quickly their derivation alongside with the general approach based on using the automorphism of varieties [8].

Let us consider the block elements introduced previously in [1–4, 7, 9]. Obviously, each of them displays the right-hand sides of inhomogeneous differential equations and the boundary conditions of the carrier taken in certain spaces $\mathbf{H}_s$ as a function in the open internal region of the carrier. The following statement is valid.

Theorem 1. The set of block elements of the boundary-value problem unambiguously solvable in certain space $\mathbf{H}_s$ and considered in the region Ω with the piece-smooth boundary $\partial\Omega$ represents a topological set with the topology having the structure of hat of the Ω-region.

In the problems of continuum mechanics, the topology in the space containing the region Ω is induced by the Euclidean space.

These are the boundary-value problem formulated in a certain region Ω with the boundary $\partial\Omega$ that are responsible for the block-element origin and analytical properties. This theorem is related to the representation of solutions of the boundary-value problems in the form of an expansion in terms of the block elements capable of being united in the elements with larger carriers leaving invariable the solution φ of the boundary-value problem.

This theorem explains the possibility of choosing a rich arsenal of every possible region admitted by the accepted topological structure and a particular boundary-value problem as the carriers of block elements. Each block-element carrier can have its own local system of coordinates, the relation of which with the local systems of carriers of neighboring blocks is controlled by a map [12, 13].

3. The results displayed below show that the dependence of the block element on the boundary-value problem is not an invariable property.

Let us consider two boundary-value problems unambiguously solvable in $\mathbf{H}_s$, for the set of partial differential equations with constant coefficients of an identical order having unknown vector functions of an identical dimension in the region Ω with the piece-smooth boundary $\partial\Omega$ [7].

The following statement takes place.

Theorem 2. The solution of one of the above boundary-value problems admits the representation in the form of the expansion in terms of block elements of another boundary-value problem considered in the region Ω with the boundary $\partial\Omega$.

4. We consider the boundary-value problem in the region Ω with the boundary $\partial\Omega$ unambiguously solvable in Hs with respect to the vector function φ_1 for the set of partial differential equations of a finite order with variable coefficients and without features.

We consider the previous boundary-value problem in the region Ω with the boundary $\partial\Omega$ with respect to the vector function of the same dimension for the set of the same partial differential equations in which the constant values are found instead of variable coefficients providing the unambiguous resolvability of the boundary-value problem in $\mathbf{H}_s$.

The following statement is valid.

Theorem 3. The solution φ_1 of the boundary-value problem with variable coefficients admits the representation of the form of the expansion in terms of the block elements of the boundary-value problem with constant coefficients.

Remark. The idea of using the “frozen-coefficient” technique, i.e., the replacement of variable coefficients in the differential equations of the boundary-value problem for constant values, was applied by Academician I. I. Vorovich when constructing the mathematical model of water resources of the Azov Sea basin [14]. However, there was no block-element method that time, and the applied technique had an a priori approximate character.

5. The possibilities of using the block-element method are even more extended due to the property presented below.

Let us consider the boundary-value problem for the set of partial differential equations of finite order unambiguously solvable in $\mathbf{H}_s$ with the maximum derivative ν in the region Ω with the boundary $\partial\Omega$ [7].

Let us designate the space of functions, which are continuously differentiated λ times with respect to all variables including the mixed ones.

It is valid as follows.

Theorem 4. An arbitrary vector function φ from $C_\lambda(\Omega)$, $\lambda > \nu$, can be represented as the expansion in terms of the block elements unambiguously solvable in a certain space $\mathbf{H}_s$ of the boundary-value problem for the vector function of the same dimension considered in the region Ω with the piece-smooth boundary $\partial\Omega$ having the maximum derivative of the order ν in the differential equations.

This theorem opens ample possibilities for the most different applications of block elements. Alongside with the results presented, these possibilities increase with using various forms of automorphism of varieties [15].

6. The factorization methods in block elements

We present a comparative analysis of two factorization methods used in applied mathematical physics and continuum mechanics [1–4]. We justify the necessity of their separation into integral and differential factorization methods, depending on the problems they are applied to. For the differential factorization method, a new algorithm for its application to boundary value problems is described that is a generalization of the results obtained in [4]. It is shown that the method reduces boundary value problems for systems of partial differential equations to an analogue of a single equation.

1. A common feature of the factorization methods that the analytic functions generated by integral transformations are regular in certain complex domains depending on the supports of the functions subject to the integral transformations. Other common features are a reduction of original problems to the study of certain functional equations and the possibility of constructing exact solutions for half-spaces. An advantage of the factorization methods is that the solution is represented in integral form. As a result, the solution can be analyzed and the input parameters of the problems can be varied to achieve the desired properties of the solution. However, the factorization methods deserve a classification according to their functions and capabilities, since they solve different problems and are based on different approaches.

2. The integral factorization method devised in [1] was based on the study and solution of integral equations or their systems given on a half-line with a difference kernel. Integral equations of these kinds, known as Wiener–Hopf equations, are generated by boundary value problems for differential equations with a change in the boundary conditions on real line or a circle. Such problems are called mixed problems. Consider integral equation

$$\int_0^\infty \mathbf{k}(x-\xi)\mathbf{q}(\xi)\,\mathrm{d}\xi = \mathbf{f}(x), \quad x>0 \qquad (6.1)$$

extended to the negative half-line by a vector function $\mathbf{e}(x)$. The Fourier transform of Eq.(6.1) yields a Wiener–Hopf vector functional equation of the form [1, 2]

$$\mathbf{K}(\alpha)\mathbf{Q}_+(\alpha) = \mathbf{F}_+(\alpha) + \mathbf{E}_-(\alpha). \qquad (6.2)$$

Here, the capital letters denote the Fourier transforms of the functions that are denoted by the corresponding lowercase letters, and the subscripts indicate the regularity of analytic functions in the upper (plus) or lower (minus) complex half-plane. The regularity property is determined by the supports of the vector function $\mathbf{f}(x)$ and $\mathbf{e}(x)$, i.e. by the positive or negative half-line. A detailed analysis of these equations and their systems was performed in [5]. A decisive role in solving the Wiener–Hopf functional equation is played by a factorization of the matrix function $\mathbf{K}(a)$ in the form of the product of two functions:

$$\mathbf{K}(\alpha) = \mathbf{K}_-(\alpha)\mathbf{K}_+(\alpha) \qquad (6.3)$$

In the general case, each element of this function is the sum of a polynomial component and the Fourier transforms of integrable functions. Here, $\mathbf{K}_+(\alpha)$ is regular in the upper half-plane, where its determinant has no zeros. The matrix function $\mathbf{K}_-(\alpha)$ has the same property in the lower half-plane. A technique for solving the functional equation or its various modifications was described in [2, 5] and is not presented here.

Note that the integral equations (6.1) defined on finite intervals rather than on a half-line are more important in applications [6, 7]. By applying functional equation Eqs.(6.2), (6.1) defined on finite intervals are reduced to Fredholm integral equations of the second kind.

3. The differential factorization method recently developed at Kuban State University [3,4] is intended for obtaining an integral representation of solutions to systems of partial differential equations with constant coefficients in domains of complex geometry. The method is designed primarily for seismology problems.

The following principles of topological algebra underline the method. The domain of the boundary value problem in question is viewed as a topological manifold with boundary. An automorphism, i.e. a topological mapping of this manifold into itself generates transformation groups that are isomorphic to some groups of nonsingular matrices. The latter generate representations of these groups described in the general case by composite special functions. The partial differential expression in the statement of the boundary value problem is treated as a differentiable mapping to the manifold of a vector field defined on the same manifold. This mapping leads to a functional equation that differs from (6.2). To ensure an automorphism, the functional equation has to be examined by the factorization method. When the special functions generated by the automorphism are invariant under the differentiable mapping, the functional equation is especially easy to study, since the boundary conditions are globally stated on coordinate surfaces. In the general case, to ensure an automorphism, we have to use local coordinates and apply a topological partition of unity.

4. Various versions of applying the factorization method to boundary value problems in various statements can be found in [3, 4] (see also the references therein). As a result of these studies, algorithms were developed for applying the differential factorization method to the study and solution of boundary value problems involving systems of partial differential equations with constant coefficients. Below, a new algorithm is demonstrated as applied to a rather general boundary value problem.

Consider a fairly general boundary value problem for a system of P partial differential equations of an arbitrary order with constant coefficients written in operator form in a convex three-dimensional domain Ω:

$$\mathbf{K}(\partial x_1, \partial x_2, \partial x_3)\varphi = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{p=1}^{P} A_{spmnk}\varphi_{p,x_1^{(m)}x_2^{(n)}x_3^{(k)}} = 0, \tag{6.4}$$

$$s = 1, 2, \dots, P, \quad A_{sqmnk} = \text{const},$$

$$\varphi = \{\varphi_1, \varphi_2, \dots, \varphi_P\}.$$

$$\varphi = \{\varphi_s\}, \quad \varphi(\mathbf{x}) = \varphi(x_1, x_2, x_3),$$

$$\mathbf{x} = \{x_1, x_2, x_3\}, \quad \mathbf{x} \in \Omega.$$

On the boundary $\partial\Omega$ we set the boundary conditions.

$$\mathbf{R}(\partial x_1, \partial x_2, \partial x_3)\varphi = \sum_{m=1}^{M_1}\sum_{n=1}^{N_1}\sum_{k=1}^{K_1}\sum_{p=1}^{P} B_{spmnk}\varphi_{p,x_1^{(m)}x_2^{(n)}x_3^{(k)}} = f_s, \tag{6.5}$$

$$s = 1, 2, \dots, \quad s_0 < P, \quad \mathbf{x} \in \partial\Omega,$$

$$M_1 < M, \quad N_1, < N, \quad K_1 < K.$$

Note that, like in the integral factorization method described above, in the differential factorization method, the boundary value problem is solved exactly if Ω is a half-space. If Ω is a convex domain, the problem is reduced to a system of normally solvable pseudodifferential equations.

To give a systematic description of the differential factorization method, we divide it into several steps.

4.1. Reduction of the differential equation to a functional equation by applying the Fourier transform.

The three-dimensional Fourier transform

$$\varphi_n(\alpha) = \iiint_{\Omega} \varphi_n(x)\, e^{i\langle\alpha\mathbf{x}\rangle} d\mathbf{x} \equiv F\varphi_n,$$

$$\varphi_m = F\varphi_m.$$

is applied to the system to reduce it to a functional equation of the form

$$\mathbf{K}(\alpha)\boldsymbol{\varphi} = \iint_{\partial\Omega} \boldsymbol{\omega},$$

$$\mathbf{K}(\alpha) \equiv -\mathbf{K}(-i\alpha_1, -i\alpha_2, -i\alpha_3) = \|k_{n\,m}(\alpha)\|. \tag{6.6}$$

Here, $\mathbf{K}(\alpha)$ is a polynomial matrix function of order P.

The components of the vector of exterior forms $\boldsymbol{\omega}$ are two-dimensional functions of the form

$$\boldsymbol{\omega} = \{\omega_s\}, \quad s = 1, 2, \ldots, P,$$

$$\omega_s = P_{12s}\, \mathrm{d}x_1 \Lambda\, \mathrm{d}x_2 + P_{13s}\, \mathrm{d}x_1 \Lambda\, \mathrm{d}x_3 + P_{23s}\, \mathrm{d}x_2 \Lambda\, \mathrm{d}x_3. \quad (6.7)$$

The exterior-form operations are defined as

$$\mathrm{d}x_1 \Lambda\, \mathrm{d}x_2 = \mathrm{d}x_1^1\, \mathrm{d}x_2^2 - \mathrm{d}x_1^2\, \mathrm{d}x_2^1,$$

$$\mathrm{d}x_1 \Lambda\, \mathrm{d}x_3 = \mathrm{d}x_1^1\, \mathrm{d}x_2^3 - \mathrm{d}x_1^3\, \mathrm{d}x_2^1,$$

$$\mathrm{d}x_2 \Lambda\, \mathrm{d}x_3 = \mathrm{d}x_1^2\, \mathrm{d}x_2^3 - \mathrm{d}x_1^3\, \mathrm{d}x_2^2.$$

Here, we introduced vectors of an arbitrary coordinate system lying in the coverings of the tangent bundle of the body surface. In a Cartesian coordinate system, we used the following notation for the tangent vectors of an arbitrary element of a covering:

$$x_1 = \left\{x_1^1, x_1^2, x_1^3\right\},$$

$$x_2 = \left\{x_2^1, x_2^2, x_2^3\right\}.$$

The coefficients of the exterior forms are given by

$$P_{12s} = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{p=1}^{P} A_{spmnk}\,(-i\alpha_1)^m \cdot (-i\alpha_2)^n \sum_{p_3=1}^{k} (-i\alpha_3)^{p_3-1}\, \varphi_{p x_3}^{(k-p_3)} e^{i\langle \boldsymbol{\alpha}\mathbf{x}\rangle},$$

$$P_{13s} = -\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{p=1}^{P} A_{spmnk}\Bigg\{(-i\alpha_1)^m \cdot (-i\alpha_3)^k \sum_{p_2=1}^{n} (-i\alpha_2)^{p_2-1}\, \varphi_{p x_2}^{(n-p_2)} e^{i\langle \boldsymbol{\alpha}\mathbf{x}\rangle} - (-i\alpha_1)^m \sum_{p_2=1}^{n}\sum_{p_3=1}^{k} (-i\alpha_2)^{p_2-1}\,(-i\alpha_3)^{p_3-1} \cdot \frac{\partial}{\partial x_3}\left(\varphi_{x_2\ \ x_3}^{(n-p_2),(k-p_3)} e^{i\langle \boldsymbol{\alpha}\mathbf{x}\rangle}\right)\Bigg\},$$

$$\begin{aligned} P_{23s} = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{p=1}^{P} A_{spmnk}\Bigg\{&(-i\alpha_2)^n \cdot (-i\alpha_3)^k \sum_{s_1=1}^{m} (-i\alpha_1)^{(s_1-1)}\, \varphi_{p x_1}^{(m-s_1)} e^{i\langle \boldsymbol{\alpha}\mathbf{x}\rangle} \\ &+ (-i\alpha_2)^n \sum_{s_1=1}^{m}\sum_{p_3=1}^{k} (-i\alpha_1)^{(s_1-1)}\,(-i\alpha_3)^{(p_3-1)} \cdot \frac{\partial}{\partial x_3}\left(\varphi_{p x_1\ \ x_3}^{(m-s_1),(k-p_3)} e^{i\langle \boldsymbol{\alpha}\mathbf{x}\rangle}\right) \\ &+ (-i\alpha_3)^k \sum_{s_1=1}^{m}\sum_{s_2=1}^{n} (-i\alpha_1)^{(s_1-1)}\,(-i\alpha_2)^{(s_2-1)} \cdot \frac{\partial}{\partial x_2}\left(\varphi_{p x_1\ \ x_2}^{(m-s_1),(n-s_2)} e^{i\langle \boldsymbol{\alpha}\mathbf{x}\rangle}\right) \\ &+ \sum_{s_1=1}^{m}\sum_{s_2=1}^{n}\sum_{s_3=1}^{k} (-i\alpha_1)^{(s_1-1)}\,(-i\alpha_2)^{(s_2-1)} \cdot (-i\alpha_3)^{(s_3-1)} \cdot \frac{\partial}{\partial x_2}\frac{\partial}{\partial x_3}\left(\varphi_{p x_1\ \ x_2\ \ x_3}^{(m-s_1),(n-s_2),(k-s_3)} e^{i\langle \boldsymbol{\alpha}\mathbf{x}\rangle}\right)\Bigg\}, \end{aligned} \quad (6.8)$$

$$\langle \boldsymbol{\alpha}\mathbf{x}\rangle = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3, \quad \varphi = \{\varphi_n\},$$
$$\alpha = \{\alpha_1, \alpha_2, \alpha_3\}, \quad \boldsymbol{\varphi} = \{\varphi_m\}.$$

4.2. Fulfillment of given boundary conditions (6.5).

To achieve this, the solution $\varphi(\partial\Omega)$ and its normal derivatives on $\partial\Omega$ taken from the boundary conditions are introduced into the representations of the exterior forms. The tangent derivatives are not taken into account. The exterior forms contain the solution φ_n and its derivatives on $\partial\Omega$. The functions or normal derivatives on the boundary are found by fitting and inverting the nonsingular matrix from boundary conditions (6.4) and are introduced into the corresponding representations of ω. The remaining functions or normal derivatives have to be found from the pseudodifferential equations obtained by transformations of the functional equations.

The following steps are to be performed to determine the remaining unknowns in the representation of the solution.

4.3. Factorization of the matrix function $\mathbf{K}(\boldsymbol{\alpha})$ in the functional equation.

Let λ_+ denote a domain containing all the zeros z_{s+}^v, $\operatorname{Im} z_{s+}^v > 0$, z_{s-}^v $\operatorname{Im} z_{s-}^v < 0$, $s\pm = 1, 2, \ldots, G_\pm$ with and with of the determinant $K(\alpha_3^\nu) = \det \mathbf{K}(\alpha_3^\nu)$, and let λ_- denote its complement to the entire plane with the boundary Γ separating the domains. The location of the contour will be specified later. By using the results of [8], the matrix function $\mathbf{K}(\alpha_3^\nu)$ can be factorized as

$$\mathbf{K}(\alpha_3^\nu) = \mathbf{K}(\alpha_3^\nu, -)\, \mathbf{K}_r(\alpha_3^\nu). \qquad (6.9)$$

Here, $\mathbf{K}(\alpha_3^\nu, -)$ is a regular matrix function in λ_- and its determinant has no zeros in this domain. The elements of the matrix function $\mathbf{K}_r(\alpha_3^\nu)$ are polynomials in α_3^ν and its determinant is independent of α_3^ν. All zeros determinant $\mathbf{K}(\alpha_3^\nu)$ with respect toα_3^ν coincide with zeros of the determinant of $\mathbf{K}(\alpha_3^\nu, -)$ that lie in λ_+,

The elements of the matrix function $\mathbf{K}^{-1}(\alpha_3^\nu, -)$ can be represented in integral form.

To derive them, we introduce the adjoint $\mathbf{K}^*(\alpha_3^\nu)$ of $\mathbf{K}(\alpha_3^\nu)$ by setting

$$\mathbf{K}^*(\alpha_3^\nu) = \|M_{pn}(\alpha_3^\nu)\|.$$

Consider a matrix function $\mathbf{K}^*(\alpha_3^\nu, m)$ of order $P-1$ obtained from $\mathbf{K}^*(\alpha_3^\nu)$ by deleting the m row and column and such that zeros ξ_n^ν of its determinant $Q(\alpha_3^\nu) = \det \mathbf{K}(\alpha_3^\nu, m)$ do not coincide with z_{s+}^v, z_{s-}^v.

The elements of the inverse matrix function are denoted by

$$[\mathbf{K}^*(\alpha_3^\nu, m)]^{-1} = \left\|Q^{-1}Q_{ps}\right\|.$$

Then the elements of the matrix function given by $\mathbf{K}^{-1}(\alpha_3^\nu, -)$, having the form

$$\mathbf{K}^{-1}\left(\alpha_{3,}^v -\right) = \left\| \begin{matrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & \ddots & & & \\ S_{m1} & S_{m2} & \dots & S_{mm} & \dots & S_{mN} \\ & & & & \ddots & \\ 0 & & & & & 1 \end{matrix} \right\|, \qquad (6.10)$$

Have an integral representation of the form

$$S_{mp}(\alpha_3^\nu) = \frac{1}{2\pi i} \oint\limits_{\Gamma_\mp} {\sum_{s=1}^{N}}' \frac{Q_{ps}(u_3) M_{sm}(u_3)\, \mathrm{d}u_3}{Q(u_3) K(u_3)(u_3 - \alpha_3^\nu)} - \left(\frac{1}{2} \mp \frac{1}{2}\right) \frac{R_{mp}(\alpha_3^\nu)}{K(\alpha_3^\nu)}, \qquad (6.11)$$

$$m \neq p,$$

$$\frac{R_{mp}(\alpha_3^\nu)}{K(\alpha_3^\nu)} = \frac{Z_{mp}(\alpha_3^\nu)}{Q(\alpha_3^\nu)K(\alpha_3^\nu)} + \sum_n \frac{Z_{mp}(\xi_n^\nu)}{Q'(\xi_n^\nu)K(\xi_n^\nu)(\xi_n^\nu - \alpha_3^\nu)},$$

$$S_{mm}(\alpha_3^\nu) = K^{-1}(\alpha_3^\nu),$$

$$\alpha_3^\nu \in \lambda_\mp, Z_{mp}(\alpha_3^\nu) = {\sum_{s=1}^{N}}' Q_{ps}(\alpha_3^\nu) M_{sm}(\alpha_3^\nu).$$

Here Γ_+ is a closed contour such that the domain λ_+ contains only the zeros z_{s+}^v, z_{s-}^v and and the domain λ_- contains only zeros ξ_n^ν. The closed contour Γ_- encloses a domain containing all zeros z_{s+}^v, z_{s-}^v, ξ_n^ν. This representation implies that the elements of $\mathrm{K}^{-1}(\alpha_3^\nu, -)$ are rational functions with their only singularities occurring at zeros z_{s+}^v, z_{s-}^v and and the term containing them $K^{-1}(\alpha_3^\nu)$, is explicitly expressed.

4.4. Reduction of the functional equation to a system of pseudodifferential equations.

The contour Γ_- is deformed so that it encloses an infinite strip with the real line and still surrounds the zeros z_{s+}^v, z_{s-}^v, ξ_n^ν.

Consider the functional equation on the real line assuming that it contains no zeros z_{s+}^v, z_{s-}^v. Otherwise, we have to use the techniques described in [6] in order to proceed to a curved real line. Obviously, the zeros z_{s+}^v and z_{s-}^v lie in the upper and lower half-planes, respectively.

In what follows, we use a local system of Cartesian coordinates $\mathbf{x}^\nu = \{x_1^\nu, x_2^\nu, x_3^\nu\}$, where the first two components lie in the tangent plane to the boundary $\partial\Omega$ and the third component lies on the outward normal. In each local coordinate system, we perform an operation that ensures an automorphism of Ω. To this end, we perform factorization (6.9) and represent functional equation (6.6) in the form

$$\boldsymbol{\varphi} = \mathbf{K}_r^{-1}(\alpha_3^\nu)\, \mathbf{K}^{-1}(\alpha_3^\nu, -) \iint\limits_{\partial\Omega} \boldsymbol{\omega}. \qquad (6.12)$$

Applying the inverse three-dimensional Fourier transform to this functional matrix equation, we require that the original vector function φ vanish for $x_3^\nu > 0$, i.e. outside Ω. Dropping the intermediate rearrangements, we obtain the relations

$$\sum_{p=1}^{P} \iint\limits_{\partial\Omega} \omega_p Z_{mp}(z_{s-}^{\nu}) = 0, \quad s- = 1, 2, \ldots, G_-, \quad Z_{mm}(\alpha_3^{\nu}) = -Q(\alpha_3^{\nu}). \tag{6.13}$$

This system consists of pseudodifferential equations.

4.5. Derivation of a representation of the solution to the boundary value problem.

In view of Section 4.2, assume that system (6.13) has been solved. Introducing the determined components into the vector of exterior forms (6.12) and applying the three-dimensional Fourier transform to $\boldsymbol{\varphi}(\alpha)$, we obtain

$$\varphi(\mathbf{x}^{\nu}) = \frac{1}{8\pi^3} \iiint\limits_{-\infty}^{\infty} \mathbf{K}_r^{-1}(\alpha_3^{\nu}) \mathbf{K}^{-1}(\alpha_3^{\nu}, -) \cdot \iint\limits_{\partial\Omega} \boldsymbol{\omega} e^{-i\langle \alpha_3^{\nu} x_3^{\nu} \rangle} d\alpha_1^{\nu} d\alpha_2^{\nu} d\alpha_3^{\nu}, \quad \mathbf{x}^{\nu} \in \Omega.$$

Due to formulas (6.11), the solution can be made more visual if we evaluate the integral with respect to by using residue theory. As a result, we have

$$\varphi(\mathbf{x}^{v}) = \frac{1}{4\pi^2} \int\limits_{-\infty}^{\infty}\int \sum_{s} e^{-i\left(\alpha_1^v x_1^v + \alpha_2^v x_2^v\right)} \cdot \left[\mathbf{K}_r^{-1}\left(i\frac{\partial}{\partial x_3^v}\right) \mathbf{T}_+\left(\alpha_1^v \alpha_2^v, z_{s+}^v\right) e^{-iz_{s+}^v x_3^v} - \mathbf{K}_r^{-1}\left(i\frac{\partial}{\partial x_3^v}\right) \mathbf{T}_-\left(\alpha_1^v, \alpha_2^v, z_{s-}^v\right) \cdot e^{-iz_{s-}^v x_3^v} \right] \mathrm{d}\alpha_1^{\nu}\, \mathrm{d}\alpha_2^{\nu}, \tag{6.14}$$

$$t_{m\pm}\left(\alpha_1^v, \alpha_2^v, z_{s\pm}^v\right) = -\sum_{p=1}^{P} \iint\limits_{\partial\Omega_\pm} \frac{\omega_p Z_{mp}(z_{s\pm}^v)}{Q(z_{s\pm}^v) K'(z_{s\pm}^v)},$$

$$\mathbf{T}_\pm = \{0, 0, \ldots, 0, t_{m\pm}, 0, \ldots, 0\}.$$

Here, the boundary $\partial\Omega$ for chosen $x_3^{\nu} < 0$, $\mathbf{x}^{\nu} \in \Omega$ is divided according to

$$\iint\limits_{\partial\Omega} \boldsymbol{\omega} = \iint\limits_{\partial\Omega_+} \boldsymbol{\omega} + \iint\limits_{\partial\Omega_-} \boldsymbol{\omega},$$

$$\iint\limits_{\partial\Omega_+} \boldsymbol{\omega} \exp(-i\alpha_3^{\nu} x_3^{\nu}) \to 0, \quad \operatorname{Im}\alpha_3^{\nu} \to \infty,$$

$$\iint\limits_{\partial\Omega_-} \boldsymbol{\omega} \exp(-i\alpha_3^{\nu} x_3^{\nu}) \to 0, \quad \operatorname{Im}\alpha_3^{\nu} \to -\infty.$$

the following rule:

In the case of a half-space or a layered medium, the pseudodifferential equations in (6.13) degenerate into algebraic ones. By inverting them, the solution is constructed in a finite form.

The problems considered in [9, 10] show that both factorization methods do not iterate and supplement each other, providing the possibility of analyzing at wider circle of problems.

7. The topological methods in block elements

There is represented the method of research and boundary problem solution for block structures. This approach introduces the topological rendering of block element's method, developed in works [1–5]. It makes the use of this method not only convenient, but it also provides the prospect of future development with involvement of profoundly developed methods of topology and theory of manifolds. Particularly it proves the possibility of wide variety of block elements' carrier forms.

Semianalytic method of block element as opposed to just computative, allowed to reveal set of earlier unknown properties of boundary problems in block structures. Thus, in works [6, 7] existence of natural viruses is revealed, in [8] the possibility of energy confinement and other process parameters that lead to their abnormal behaviour is discovered. We can continue with examples [9]. The exposition of topological method, though it is a repetition of algorithm of block elements method, brings it nearer to the new potentials in accordance with use of deep theoretical developments in topology.

1. Lets believe, that we examine linear boundary problem for the differential equation system in partial derivatives for the block structure, consisting of blocks, which occupy three-dimensional area Ω_b $b = 1, 2, \ldots, B$, whose deformable environments have multitype physical and mechanical characteristics. The blocks contact with each other, one part of their bounds can be loose. The blocks can be limited and unlimited, they can occupy as simply connected, so multiply connected areas with piecewise-smooth boundary $\partial\Omega_b$.

Lets introduce several topologies. The first one connects with Ω_b areas, occupying the blocks irrelatively to the boundary problem. We suppose, that block structure consists of contact blocks and presents all-in-one, it may also be multiply connected. Examining block areas in metric space, lets bring into each of them topology, induced with open environs $\upsilon_{\nu b}$, $\nu = 1, 2, \dots, \nu_b$ [10, 11]. The block environs can get crushed or consolidated. The largest open environ in the block is $\upsilon_b = \cup \upsilon_{\nu b}$.

It is a consolidation of all open environs. It represents the interior of the block, and its locking $\bar{\upsilon}_b$ gives the block with bounds. Topological spaces, constructed in each block, are subspaces T_{1b} of topological space of the whole block structure T_1. Lets accomplish compactification of space T_1, by adding the environs of infinitely remote point, if the block structure consist it.

Lets enter into consideration functions from the space $\mathbf{H}_s$ in every open space $\upsilon_{\nu b}$. Linear normed space induces topology, for example, with open functions balls. Lets examine set of functions of $\mathbf{H}_s$ in every open environ $\upsilon_{\nu b}$, as in a carrier, and construct topological structure, taking as set the open balls

$$\|\varphi\|_{\mathbf{H}_s} < \varepsilon.$$

Thus the open set of functions $\Upsilon_{\nu b}$ is formed on every open set $\upsilon_{\nu b}$. The totality of open environs $\Upsilon_{\nu b}$ forms in the areas $\Omega_{\nu b}$ topological structure of subspace T_{2b}, which is a part of topological structure T_2, including open sets of all blocks. According to environs T_2 construction the spaces T_1 and are isomorphic.

Since topological space T_1 is regular in construction, it with its every subspace allows partition of unity. Lets perform the unity partition of compacted topological space T_1, and therefore of every subspace T_{1b}, with non-intersecting connected open covering, which we will mark as usual for the sake of brevity as $\upsilon_{\lambda b}$, $\lambda = 1, 2, \dots, \lambda_b$. It shows that in virtue of isomorphism the partition of space unity T_1 involves equivalent partition of space unity T_2. Let's construct topological manifold M_{1b} in topological space T_{1b}, for this we will enter local systems of coordinates, maps and atlas in every covering $\upsilon_{\nu b}$. Their consolidation results multifold M_1. Let's name open coverings $\upsilon_{\lambda b}$ as interiors of manifolds M_{1b} and their closings $\bar{\upsilon}_{\lambda b}$ as orientable manifolds with edge M_{1b}, after entering of local system of coordinate and tangent bundle of bounds. We got a totality of oriented infinitely smooth multifold M_{1b} with edge. In virtue of isomorphism the functions, forming T_2 form also multifold M_2 and M_{2b}. We can examine them as objects of topological space T_2, so as the functions on multifold $M_1 = \cup M_{1b}$. Let's indicate through Θ the additional areas Ω till the whole space R^3 as $\Theta = R^3 \backslash \Omega$, which does not contain carriers of block structure. Let's perform the covering areas Θ with open areas $\theta_{\mu r}$, which may contact with some multifolds at the place, where their bounds is loose and which we will name as null multifolds $\upsilon_{\lambda b}$. As a result the covering of the whole space R^3 will be performed with open non-intersecting multifolds.

2. Lets enter into consideration a boundary problem for the system of differential equation in partial derivative at the area, occupying with block structure

$$\mathbf{K}_b(\partial x_1, \partial x_2, \partial x_3)\varphi_b = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{p=1}^{P} A^b_{spmnk}\varphi_{bp,\overset{(m)}{x_1}\overset{(n)}{x_2}\overset{(k)}{x_3}} = g_b(\mathbf{x}), \tag{7.1}$$

$$s = 1, 2, \dots, P_b, \quad A^b_{sqmnk} = \text{const},$$
$$\varphi_b = \{\varphi_{b1}, \varphi_{b2}, \dots, \varphi_{bP}\}, \quad b = 1, 2, \dots, B.$$
$$\varphi = \{\varphi_s\}, \quad \varphi(\mathbf{x}) = \varphi(x_1, x_2, x_3),$$
$$\mathbf{x} = \{x_1, x_2, x_3\}, \quad \mathbf{x} \in \Omega_b.$$

The following boundary conditions are set on the common contacting bounds

$$\mathbf{R}_b(\partial x_1, \partial x_2, \partial x_3)\varphi_b + \mathbf{R}_d(\partial x_1, \partial x_2, \partial x_3)\varphi_d = \sum_{m=1}^{M_1}\sum_{n=1}^{N_1}\sum_{k=1}^{K_1}\sum_{p=1}^{P}\Big[B^b_{spmnk}\varphi_{bp,\overset{(m)}{x_1}\overset{(n)}{x_2}\overset{(k)}{x_3}} + B^d_{spmnk}\varphi_{dp,\overset{(m)}{x_1}\overset{(n)}{x_2}\overset{(k)}{x_3}}\Big] = f_{bds}, \tag{7.2}$$

$$s = 1, 2, \dots, s_{b0} < P, \quad \mathbf{x} \in \partial\Omega_b \cap \partial\Omega_d,$$
$$M_1 < M, \quad N_1, < N, \quad K_1 < K,$$
$$b,\, d = 1, 2, \dots, B.$$

In the case, if the area Ω_d is null area, only the term with index, b the loose bounds, remains in the formula under the sign of sum.

The boundary problem studies in spaces of temperate generalized function $\mathbf{H}_s(\Omega)$, described [1, 2].

Let's introduce in consideration a Cartesian product of topological space $T_1 \times T_2$. Let's undergo its mapping concerning the rule: T_1 is mapped identically to itself; mapping of T_2 is introduced by the form $\mathbf{K}_b(\partial x_1, \partial x_2, \partial x_3)\varphi_b$, which transmits a vector φ_b, from M_{2b} to an assigned vector g_b to M_{2b} concerning (7.2).

For solution of a boundary problem it is necessary to find an image of this mapping.

2.1. Start with a setting of a case, when the ratios of differential form (7.1) are constant in every block. Proceeding in a space R^3 to functions from T_2, belonging to $\mathbf{H}_s(\Omega)$, to topological dual of Fourier – images, receive the relations, which are called the functional equation type (7.2),

$$\mathbf{K}_b(\alpha)\boldsymbol{\varphi}_b = \iint\limits_{\partial\Omega_b} \boldsymbol{\omega}_b - \mathbf{G}_b\,(\alpha),$$

$$\mathbf{G}_b\,(\alpha) = \iiint\limits_{\Omega} \mathbf{g}_b(\mathbf{x}) \exp i\,\langle\alpha\mathbf{x}\rangle\ \mathrm{d}x_1x_2x_3,$$

$$\mathbf{K}_b\,(\alpha) \equiv -\mathbf{K}_b\,(-i\alpha_1, -i\alpha_2, -i\alpha_3) = \|k_{bnm}(\alpha)\|\,,$$

$$\alpha\mathbf{x} = \alpha_1x_1 + \alpha_2x_2 + \alpha_3x_3, \quad b = 1, 2, \ldots, B.$$

where the symbols of this work are saved. In process of fulfilled buildings the Stokes' integral was used on multifold with edge, which led to appearing of exterior form $\boldsymbol{\omega}_b$. From developed functional equations we find vectors representation φ_b, involving unknown values on blocks' boundaries. For finding the solutions of boundary values φ_b at conjugated or free boundaries of blocks' region $\partial\Omega_b$ it is necessary to unify the boundaries of contacts or adjacent coverings $\bar{\upsilon}_b$ and $\bar{\upsilon}_d$, or adjacent coverings $\bar{\upsilon}_d$ and $\bar{\theta}_d$ if a block has a free boundary. For this purpose a factor is built – a space topology $T_1 \times T_2$, which fulfills a factor of equivalent block boundaries or the same, equivalent boundaries of topological closed-sets, under condition (7.2). Let's mark, that in this case in space T_1 an uprising of coverings by uniting them by the boundary, and in T_2 space a pseudodifferential equation is formed, which carries out the union of the covering elements of this topological space. This condition is far more complicated than the traditionally stated examples of factor-spaces in literature of topology [11], but only after its realization a factor-topology in our case is formed. A suitable algorithm of conducting this process is described in [2]. Let's suppose that 2 blocks $\bar{\upsilon}_b$ and $\bar{\upsilon}_d$ are in contact. Let's examine only a part of boundary of their interaction irrespective to the contacts with other blocks. Then the procedure, which describes the building of resolving interactions for solving primary boundary problem, includes the following actions:

– analyzing the multifold with edge M_{2b} and M_{2d} in local coordinate axes [10];

– a choice of the most effective curvilinear coordinate system for fulfillment of automorphism [12, 13];

– fulfillment of a differential factorization of matrix-function $\mathbf{K}_b\,(\alpha)$ and $\mathbf{K}_d\,(\alpha)$ [14];

– computing the Leray residue form [2];

– building of pseudodifferential equations [2];

– extracting from pseudodifferential equations by the demanded condition (7.2) of integral equations [2, 13, 14];

– the solution of integral equations;

– adding the found solutions into the integral representation of a solution of boundary problem

$$\varphi_b = \mathbf{F}^{-1}\mathbf{K}_b^{-1}(\alpha)\left[\iint\limits_{\partial\Omega_b} \boldsymbol{\omega}_b - \mathbf{G}_b\,(\alpha)\right];$$

– operator $\mathbf{F}^{-1}$ of Fourier transforms.

In this case after the enlargement of manifold M_{1d} and M_{1b} by uniting into $M_{1b} \cup M_{1d}$, building of isomorphic manifolds $M_{2b} \cup M_{2d}$ will follow.

2.2. The case of variable coefficients in differential form (7.1) is different from the case, which is discussed above, that the covering is dictated by the features of its coefficients. it should be as small in size, that analyzing at it differential form (7.1) could be related to a category, which has constant coefficients. Than every such covering becomes the block element, but which has the coefficients. In this case at the boundary of such blocks the boundary conditions interface solutions should be formulated, similarly (7.2), dictated by the demand of software smoothness of solutions, it is possible, theirs differential quotients or gradients and other forms at the boundary. After that, all actions for such block structure for the given boundary problem, which are stated in the previous paragraph.

2.3. In case of nonlinear boundary problem a research and its solution can be fulfilled by using the Newton-Kantorovich's method [15], which demands for its realization at each step a converse of some linear not uniform boundary problems with variable coefficients, which, it is shown in previous paragraphs, is feasible for the method.

Therefore, an offered method can be researched a wide range of boundary problems from different fields. It should be noticed, that usage of this method allows building of analytical representation of boundary problem solution, and it is of extreme importance, for example, for analyzing a wave processes and revealing of different irregular conditions in a multiparameter processes.

Note 1. The simplified schemes of multifold buildings are stated at 2.1 in terms that the blocks have one map. Without any effort using this scheme a research can be fulfilled also in the cases, when there are several maps. In this case it is necessary to "increase" the quantity of blocks, putting for each one map. In this case it is necessary to form additional conditions of the type (7.2), providing the continuation of solutions from one block to another concerning the demanding dictating by the boundary problem conditions of persistence.

Note 2. Application to topological approach in the method of block element allowed setting important property of possibility of wide chose of its carriers which can be absolutely arbitrary open sets. For all these cases exists an algorithm providing the process of getting the answer bounding sum.

Note 3. It is perfectly clear that given block structure can contain different types of heterogeneity as cracks, inclusions and vesicle. In his case block elements can be built with the method of relative (virtual) division in block structure without crossing border of heterogeneity. In the case when three-dimensional block structure contains deformed blocks of smaller sizes, for example, plates or shells coupling of such elements has its own peculiarity and the algorithm stated above cannot be used.

8. The topological approach in mechanical conception

The theoretical foundations for prognosis of seismicity are based on the concept of evaluation of the concentration of stresses from the mechanical interaction of lithospheric plates subjected to external effects of various natures and performing a slow drift. There are different approaches to its implementation in some domestic and foreign studies. However, their features considerably simplify the problems under consideration and allow application of a very simple mathematical apparatus. Such an approach as a whole cannot envelope a tremendous variety of processes in deep layers of the Earth and on its surface, as well as the properties of deformed media, of which lithospheric plates consist, and the types of faults. We note that the information (in many aspects relative to the depth properties) is extremely poor and unknown in some cases. In connection with this, an approach to modeling the processes of the behavior of lithospheric plates that would make it possible to interact with the model in a dialog mode as is required for involving newly revealed information into it is necessary.

For example, such information can involve the presence of nonuniformities of lithospheric plates; the refinement of the types of their faults, if they are through or of a limited depth; and the presence of internal cavities and cracks in lithospheric plates. The information on the properties of the contact of lithospheric plates with the Earth's upper mantle, asthenosphere, and questions on the possible effects of the sources of stresses of all types on lithospheric plates, involving those from the daylight – wind and sedimentary, radiation, and electromagnetic ones. Attempts to take these factors into account, which were performed in certain approaches, do not make it possible to perform it as a whole, which leads to their separation and investigation of separate problems for each of them. This leads not only to loss of accuracy but also to loss of important properties of the behavior of solutions, boundaries, and things associated with the manifestation of natural viruses [1–3]. The latter were found only in connection with investigations of the influence of several factors on the solutions of boundary problems considered jointly.

In connection with the aforesaid, the block element method, which has a topological base [4] and has been successfully developed and applied at the Southern Scientific Center of the Russian Academy of Sciences and at Kuban University, is the most appropriate for investigation of the behavior of lithospheric plates. Implementation of the mechanical concept is demonstrated below by the example of the simplest model of lithospheric plates. The mechanical concept of the evaluation of the seismicity of territories is based on revealing the stress concentration zones in lithospheric plates as one more precursor of seismicity, by which we can judge the possible consequences of seismic events, the places of their arrangement, and, in some situations, their probable arrival times.

1. Let us consider separately the topological approach in the theory of block structures in the presence of block elements of various dimensions. In contrast to block structures with blocks of the same dimensionality as considered in [4, 5], this case has its own specifics. It can be investigated by different approaches. As a rule, these are the cases in which the three-dimensional deformed bodies are in contact with two-dimensional ones, for example, with plates or shells. Coatings of three-dimensional bodies by the shells, the presence of technological inclusions from the plates in three-dimensional bodies, etc., are referred to such contacts. The coating can be multilayered. Coatings can be partially cleaved and contain cracks. Such a situation occurs in lithospheric plates, which contain faults, both internal and external. An example of tectonic faults on the territory of Krasnodar region is presented. Cracked coatings are also possible in aviation materials, where the levels of admissible defects of airplanes, which allow the continuation of their safe service, are determined. In materials science, theories are developed that explain the durability of a ground surface of metals by the presence of the surface tension modeled by a thin coating. The problems of studying the durability of such objects, with the presence of cracks in them, appear in the theory of nanocoatings.

When studying topologically a block structure consisting of the above-described two-dimensional and three-dimensional blocks, two approaches are possible. The first one involves the priority topological investigation of each block structure, two-dimensional and three-dimensional, separately by the method described in [4–7] allowing for the presence of all nonuniformities, cracks, and faults. Them an operation is performed, which is called the construction of the factor-topology and consists in identifying the two-dimensional boundary of three-dimensional block element with the middle surface of the two-dimensional coating. In such a manner, we construct the pseudodifferential equations and the integral equation for the construction of all boundary conditions of the boundary problem under consideration.

The second approach consists in the preliminary construction of the factor-topology of two block elements, the three-dimensional and two-dimensional ones, with subsequent investigation of a new topological object, which contains nanodimensional components and the same various-dimensional boundaries. The investigation in the second way requires correctly taking into account of all the features of such a topological object. This is especially referred to constructing the tangential exfoliation of the boundary and introducing local coordinate systems, the maps, and the atlas of the manifold.

2. As an example, let us consider the boundary problem for a wafer as the simplest model giving a various-dimensional block structure having contact with a three-dimensional substrate. Let us accept a plate consisting of different types of horizontally contacting fragments, which can also be cracked, and situated in the deformed half-space. In particular, this is the simplest model of lithospheric plates modeled by Kirchhoff plates. It consists of the parts of horizontally oriented plate-blocks and contains arbitrary geometric faults. Currently, the experimental data relative to the motions of lithospheric plates are obtained using high-precision GPS/GLONASS receivers [8]. Based on this model, we will consider such a plate with faults as a two-dimensional manifold with an edge. Let us denote the regions occupied by the plate as Ω. Let us divide the plate into blocks starting from the requirement of retaining the uniformity and constancy of properties in each block. In addition, let us perform the division into the blocks over the faults or cracks even if the crack intersects the single-type block. Let B be the number of blocks obtained after such division. In this case, we have $\Omega = \cup\Omega_b$, $b = 1, 2, \ldots, B$. The boundaries of blocks $\partial\Omega_b$ will be of different types. A part $\partial\Omega_{b1}$ of each boundary $\partial\Omega_b$ can provide a rigid contact with a neighboring block, another part $\partial\Omega_{b2}$ can be free of stresses and bends, and the third part $\partial\Omega_{b3}$ can be the crack edge, i.e. in the general case, $\partial\Omega_b = \cup\partial\Omega_{br}$, $r = 1, 2, 3$.

Let us retain the notations of motions conventional in plate theory $\mathbf{u} = \{u_1, u_2, u_3\}$ and mechanical parameters [9, 10]. Below, u_1, u_2, are the motions of plate points along the horizontal direction of the middle surface and u_3 is the motion along the normal to it. In this case, the expression for the differential component of the operator

$$\mathbf{R}_b\left(\partial x_1, \partial x_2\right)\mathbf{u}_b - \varepsilon_{5b}\mathbf{g}_b = \varepsilon_{5b}\mathbf{t}_b.$$

Has the form

$$\mathbf{R}_b\left(\partial x_1, \partial x_2\right)\mathbf{u}_b = \begin{Vmatrix} \rho_{11} & \rho_{12} & 0 \\ \rho_{21} & \rho_{22} & 0 \\ 0 & 0 & \rho_{33} \end{Vmatrix}.$$

Here,

$$\rho_{11} = \left(\frac{\partial^2}{\partial x_1^2} + \varepsilon_{1b}\frac{\partial^2}{\partial x_2^2} + \varepsilon_{4b}\right)u_{1b},$$

$$\rho_{22} = \left(\frac{\partial^2}{\partial x_2^2} + \varepsilon_{1b}\frac{\partial^2}{\partial x_1^2} + \varepsilon_{4b}\right)u_{2b},$$

$$\rho_{33} = \left(\varepsilon_{3b}\left(\frac{\partial^4}{\partial x_1^4} + 2\frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2} + \frac{\partial^4}{\partial x_2^4}\right) - \varepsilon_{4b}\right)u_{3b},$$

$$\rho_{12} = \left(\varepsilon_{2b}\frac{\partial^2}{\partial x_1 \partial x_2}\right)u_{2b},$$

$$\rho_{21} = \left(\varepsilon_{2b}\frac{\partial^2}{\partial x_1 \partial x_2}\right)u_{1b},$$

$$\mathbf{R}_b(-i\alpha_1, -i\alpha_2)\mathbf{U}_b = -\begin{Vmatrix} \xi_{11} & \xi_{12} & 0 \\ \xi_{21} & \xi_{22} & 0 \\ 0 & 0 & \xi_{33} \end{Vmatrix}. \quad (8.1)$$

Here,

$$\xi_{11} = (\alpha_1^2 + \varepsilon_{1b}\alpha_2^2 - \varepsilon_{4b})U_{1b},$$

$$\xi_{22} = (\alpha_2^2 + \varepsilon_{1b}\alpha_1^2 - \varepsilon_{4b})U_{2b},$$

$$\xi_{33} = -(\varepsilon_{3b}(\alpha_1^2 + \alpha_2^2)^2 - \varepsilon_{4b})U_{3b},$$

$$\xi_{12} = \varepsilon_{2b}\alpha_1\alpha_2 U_{2b}, \quad \xi_{21} = \varepsilon_{2b}\alpha_1\alpha_2 U_{1b},$$

$$\mathbf{U} = \mathbf{F}_2\mathbf{u}, \quad \mathbf{G} = \mathbf{F}_2\mathbf{g}, \quad b = 1, 2, \ldots, B.$$

Here,

$$\varepsilon_{1b} = \frac{1}{2}(1 - \nu_b), \quad \varepsilon_{2b} = \frac{1}{2}(1 + \nu_b),$$

$$\varepsilon_{3b} = \frac{h_b^2}{12},$$

$$\varepsilon_{4b} = \omega^2\rho_b\frac{1 - \nu_b^2}{E_b}, \quad \varepsilon_{5b} = \frac{1 - \nu_b^2}{E_b h_b}, \quad (8.2)$$

$$g_{1b} = \mu_b\left(\frac{\mathrm{d}u_{1b}}{\mathrm{d}x_3} + \frac{\mathrm{d}u_{3b}}{\mathrm{d}x_1}\right),$$

$$g_{2b} = \mu_b\left(\frac{\mathrm{d}u_{2b}}{\mathrm{d}x_3} + \frac{\mathrm{d}u_{3b}}{\mathrm{d}x_2}\right),$$

$$g_{3b} = \lambda_b\left(\frac{\mathrm{d}u_{1b}}{\mathrm{d}x_1} + \frac{\mathrm{d}u_{2b}}{\mathrm{d}x_2} + \frac{\mathrm{d}u_{3b}}{\mathrm{d}x_3}\right) + 2\mu_b\frac{\mathrm{d}u_{3b}}{\mathrm{d}x_3},$$

$$x_3 = 0.$$

Notations are accepted for the plates: λ and μ are the Lamé parameters, ν is the Poisson coefficient, E is the Young modulus, h is the thickness, ρ is the density, ω is the oscillation frequency, $\mathbf{g}_b = \{g_{1b}, g_{2b}, g_{3b}\}$ and $\mathbf{t}_b = \{t_{1b}, t_{2b}, t_{3b}\}$ are the vectors of contact stresses and external powers acting along axis x_3 in region Ω_b, and $\mathbf{F}_2 \equiv \mathbf{F}_2(\alpha_1, \alpha_2)$ is the two-dimensional Fourier transform operator.

Variable boundary conditions are dictated by the type of the parts of boundaries of each block. For example, with the accepted notations, the boundary conditions for the case of hinged opening in the contact zone, i.e., free rotation at the boundary around axis x_1, has the form

$$M = -D\left(\frac{\partial^2 u_3}{\partial x_1^2} + \nu\frac{\partial^2 u_3}{\partial x_2^2}\right) = 0, \quad D = \frac{Eh^2}{12(1 - \nu^2)}. \quad (8.3)$$

For the case when the plate edges are allowed to shift freely along axis x_3, the boundary condition is

$$Q = -D\left(\frac{\partial^3 u_3}{\partial x_2^3} + (2 - \nu)\frac{\partial^3 u_3}{\partial x_1^2 \partial x_2}\right) = 0. \quad (8.4)$$

In the case of rigid fixing, we should require that the shifts in axial directions would equal zero:

$$u_1 = 0, \quad u_2 = 0, \quad u_3 = 0. \quad (8.5)$$

To forbid the turn of the middle plane around axis x_1, we should require fulfillment of the condition

$$\frac{\partial u_3}{\partial x_2} = 0. \quad (8.6)$$

Expressions for the normal and tangential components of stresses at the boundary are given by expressions, respectively,

$$N_{x_2} = \frac{E}{1-\nu^2}\left(\frac{\partial u_2}{\partial x_2} + \nu\frac{\partial u_1}{\partial x_1}\right),$$

$$T_{x_1x_2} = \frac{E}{2(1+\nu)}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right).$$

As the deformed base of the substrate, which is described by boundary problem (8.1) with coating plates arranged on it, we can accept different models. These can be deformed half-space, the layer, and the multilayered half-space including the anisotropic one, viscoelastic media. In all listed cases, the ratios between the stresses on the surface of the layered medium g_{kb}, $k=1,2,3$ and motions u_k, $k=1,2,3$ have form (8.2) with properties

$$\mathbf{u}(x_1,x_2,x_3) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int \mathbf{K}(\alpha_1,\alpha_2,x_3) \cdot \mathbf{G}(\alpha_1,\alpha_2)e^{-i\langle\alpha,x\rangle}\,\mathrm{d}\alpha_1\,\mathrm{d}\alpha_2, \quad (8.7)$$

$$\langle\alpha,x\rangle = \alpha_1x_1+\alpha_2x_2,$$

$$\mathbf{K} = \|K_{mn}\|, \quad m,n=1,2,3,$$

$$\mathbf{K}(\alpha_1,\alpha_2,0) = O(A^{-1}),$$

$$A = \sqrt{\alpha_1^2+\alpha_2^2} \to \infty.$$

$K_{ks}(\alpha_1,\alpha_2,x_3)$ are the analytical functions of two complex variables α_k, particularly, meromorphic ones; their numerous parameters are presented in [11, 12]. These relations are called the dominant functions.

If equations that describe the behavior of the base medium are known, the elements of the function-matrix $\mathbf{K}(\alpha_1,\alpha_2,0)$ can be calculated. If there are no such equations, influence functions can be obtained experimentally.

3. As an example, let us consider the scalar case of vertical oscillations of the plate. In this case, functional equation (8.1) of the boundary problem for this case, which s presented for the above-described plate as for the manifold with the edge, is split for each block and given by relationship [13]

$$\mathbf{R}_b(-i\alpha_{1b},-i\alpha_{2b})U_{3b} \equiv (\varepsilon_{3b}(\alpha_{1b}^2+\alpha_{2b}^2)^2-\varepsilon_{4b})U_{3b} = -\int_{\partial\Omega_b}\omega_b + \varepsilon_{5b}\mathbf{F}_2(g_{3b}+t_{3b}), \quad (8.8)$$

$$b = 1,2,\ldots,B.$$

Here, ω_b is the external form that participates in the representation and has the form

$$\omega_b = \varepsilon_{3b}e^{i\langle\alpha,x\rangle}\Biggl\{-\Biggl[\frac{\partial^3u_{3b}}{\partial x_2^3} - i\alpha_2\frac{\partial^2u_{3b}}{\partial x_2^2} - \alpha_2^2\frac{\partial u_{3b}}{\partial x_2} + i\alpha_2^3u_{3b} + 2\frac{\partial^3u_{3b}}{\partial x_1^2\partial x_2} - 2i\alpha_2\frac{\partial^2u_{3b}}{\partial x_1^2}\Biggr]\,\mathrm{d}x_1 + \Biggl[\frac{\partial^3u_{3b}}{\partial x_1^3} - i\alpha_1\frac{\partial^2u_{3b}}{\partial x_1^2} - \alpha_1^2\frac{\partial u_{3b}}{\partial x_1} + i\alpha_1^3u_{3b}\Biggr]\,\mathrm{d}x_2\Biggr\},$$

$$\omega_b = \varepsilon_{3b}e^{i\langle\alpha,x\rangle}\Bigl\{-\Bigl[i\alpha_2MD^{-1} - QD^{-1} - (\alpha_2^2+\nu\alpha_1^2)\frac{\partial u_{3r}}{\partial x_2^r} + i\alpha_2\left[\alpha_2^2+(2-\nu)\alpha_1^2\right]u_{3r}\Bigr]\Bigr\}\,\mathrm{d}x_1.$$

As is mentioned above, the block boundary can contact differently with neighboring blocks or be free. This property should be introduced into the presentation of the pseudodifferential equation.

To construct it, the roots of the coefficient of functional equation (8.8) are found, and then the automorphism requirement id fulfilled as applied to the functional equation, and the Leray residue form is calculated.

Let us admit that one of the parts of the boundary of block Ω_{br} is a straight line. In this case, the group of pseudodifferential equations constructed based on this part takes the form

$$\mathbf{F}_1^{-1}(\xi_1^r)\Biggl\langle -\int_{\partial\Omega_{br}}\Bigl\{i\alpha_{21-}D^{-1}M_r - D^{-1}Q_r - (\alpha_{21-}^2+\nu\alpha_1^2)\frac{\partial u_{3r}}{\partial x_2^r} + i\alpha_{21-}\left[\alpha_{21-}^2+(2-\nu)\alpha_1^2\right]u_{3r}\Bigr\}e^{i\alpha_1^rx_1^r}\,\mathrm{d}x_1^r + \int_{\partial\Omega_b\backslash\partial\Omega_{br}}\omega_b + \varepsilon_{5b}\mathbf{F}_2(g_{3b}+t_{3b})\Biggr\rangle = 0,$$

$$\alpha_2 = \alpha_{21-}, \quad \xi_1^r \in \partial\Omega_{br},$$

$$\mathbf{F}_1^{-1}(\xi_1^r)\left\langle -\int\limits_{\partial\Omega_{br}} \left\{ i\alpha_{22-}D^{-1}M_r - D^{-1}Q_r \right.\right.$$
$$- (\alpha_{22-}^2 + \nu\alpha_1^2)\frac{\partial u_{3r}}{\partial x_2^r}$$
$$\left. + i\alpha_{22-}\left[\alpha_{22-}^2 + (2-\nu)\alpha_1^2\right] u_{3r} \right\} e^{i\alpha_1^r x_1^r}\, \mathrm{d}x_1^r$$
$$\left. + \int\limits_{\partial\Omega_b \setminus \partial\Omega_{br}} \omega_b + \varepsilon_{5b}\mathbf{F}_2(g_{3b}+t_{3b}) \right\rangle = 0,$$
$$\alpha_2 = \alpha_{22-}, \quad \xi_1^r \in \partial\Omega_{br}.$$

Here, $\mathbf{F}_1^{-1}$ are the backward operators to the Fourier one-dimensional representations. It should be accepted in integrands that

$$\alpha_{21-} = -i\sqrt{(\alpha_1^r)^2 - \sqrt{\varepsilon_{4b}/\varepsilon_{3b}}},$$
$$\alpha_{22-} = -i\sqrt{(\alpha_1^r)^2 + \sqrt{\varepsilon_{4b}/\varepsilon_{3b}}}$$

respectively. If the boundary is not straight, we can consider that a small zone of this boundary is considered. Other groups of pseudodifferential equations for other segments of the boundary are shaped similarly. The property of the main operator to contain boundary conditions of all types, which the plate can have during vertical vibrations and for which the analytical expressions are given by relations (8.3)–(8.6), is a characteristic property. Writing all pseudodifferential equations for each segment of the boundary and for each block, introducing the corresponding boundary conditions into them, and solving the integral equations derived from pseudodifferential equations, we derive the representation of solutions in each plane block from relations.

$$u_{3b} = \mathbf{F}_2^{-1}\left[\mathbf{R}_b(-i\alpha_{1b}, -i\alpha_{2b})\right]^{-1} \cdot \left\langle -\int\limits_{\partial\Omega_b} \omega_b + \varepsilon_{5b}\mathbf{F}_2(g_{3b}+t_{3b}) \right\rangle. \quad (8.9)$$

The found representation u_{3b} (8.9) of the two-dimensional block structure is identical to u_{3b} (8.7) at $x_3 = 0$ of the three-dimensional block structure, and as a result, the integral equation for determining the contact stresses between the coating and the substrate, which carries information on the stress concentration in the lithospheric plate from vertical effects, is obtained. The presented scalar scale is transferred to the vector case, which is described by the first two equations in (8.1), without any difficulty. In total, with the solution of the complete boundary problem for set of equations (8.1), the possibility appears to evaluate the stress concentration in the models of lithospheric plates based on the monitoring information involving the GPS/GLONASS receivers. A more exact model of the territory is obtained with the use of deformed three-dimensional lithospheric plates arranged horizontally on a deformed base and with applying the approaches described in [15].

9. About hidden defects in the bodies with coverings

The method of hidden flaws in covers investigation is being developed, a resumptive topologic approach in dynamic boundary problems [1, 2] in case of statistic problems. The tense state of strain of block structure is being investigated, that consists of aflat located polytypic blocks contacting at the borders between themselves. This block structure is located on the surface of three-dimensional linearly deformable substrate. The considered block structures are under the vertical static external action. This state is typical for lithosphere plates and also nanomaterials and articles made of structural materials. The investigation of the tense state of strain of lithosphere plates state in static conditions allows to receive information about the territory seismicity character. The topologic approach makes it possible to consider simultaneously bodies with covers having hidden flaws unobserved by sight as opposed to those, for example, considered in [3]. By the example of block structure consisting of two polytypic contact planes on the three-dimensional deformable substrate the case of hidden flaw existence was considered that preceded the demolition in this zone. Let us note that it is impossible to get the static case by solving the analogical boundary problem concerning harmonic oscillations using simple passage to the limit to vanishing oscillation frequency [1]. Static problems for covers with flaws can disclose seismicity growth in heightened danger of earthquakes territories and also promotes, on all due conditions, the passing of “quiet earthquakes”. The detailed analysis of the approach was made for significant in supplements case of two cover fragments in the form of half planes contact that occur the most in heightened danger of earthquakes territories.

1. Without going into details of solving of topologic boundary problems and factorization approaches stated in [1, 2, 4–9] let us show the determining equations for block structure consisting of two-dimensional cover fragments on three-dimensional substrate retaining the description of works [1, 2]. The Kirchhoff equation for some block $b$cover $b = 1, 2, \ldots, B$ occupying the area Ω_b with boundary $\partial\Omega_b$ during vertical static actions by the help of stresses t_{3b} has the form

$$\mathbf{R}_b(\partial x_1, \partial x_2)u_{3b} - \varepsilon_{5b}(g_{3b} + t_{3b}) \equiv \varepsilon_{3b}\left(\frac{\partial^4}{\partial x_1^4} + 2\frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2} + \frac{\partial^4}{\partial x_2^4}\right)u_{3b} - \varepsilon_{5b}(g_{3b} + t_{3b}) = 0,$$

$$\mathbf{R}_b\left(\partial x_1, \partial x_2\right)u_{3b} - \varepsilon_{5b}g_{3b} = \varepsilon_{5b}t_{3b},$$

$$\mathbf{R}_b(-i\alpha_1, -i\alpha_2)U_{3b} \equiv R_b(-i\alpha_1, -i\alpha_2)U_{3b} \equiv \varepsilon_{3b}(\alpha_1^2 + \alpha_2^2)^2 U_{3b},$$

$$U_{3b} = \mathbf{F}_2 u_{3b}, \quad G_{3b} = \mathbf{F}_2 g_{3b}, \quad b = 1, 2, \ldots, B$$

$$\varepsilon_{1b} = \frac{1}{2}(1 - \nu_b), \quad \varepsilon_{2b} = \frac{1}{2}(1 + \nu_b),$$

$$\varepsilon_{3b} = \frac{h_b^2}{12}, \quad \varepsilon_{5b} = \frac{1 - \nu_b^2}{E_b h_b}$$

Here the plates have the following markings: ν – Poisson's ratio, E – Young's modulus, h – thickness, g_{3b}, t_{3b} – contact voltage and external pressure value that action longwise axis x_3 in area Ω_b, and $\mathbf{F}_2 \equiv \mathbf{F}_2(\alpha_1, \alpha_2)$, $\mathbf{F}_1 \equiv \mathbf{F}_1(\alpha_1)$ are accordingly the two-dimensional and one-dimensional Fourier-transform operator.

In the local coordinate system $x_1x_2x_3$ with the plane x_1x_2 that coincides with middle plate flatness, axis ox_3 directed via the normal line to plate by the axis ox_1 directed tangentially to the break boundary by the axis ox_2 – via the normal to it's boundary, the end conditions can be assigned by any two of the four following correlations, and namely, in the form of vertical dislocation on the boundary

$$u_{3b} = f_1(\partial\Omega_b) \tag{9.1}$$

middle plate rotating round the axis x_1,

$$\frac{\partial u_{3b}}{\partial x_2} = f_2(\partial\Omega_b) \tag{9.2}$$

bending moment on the boundary

$$M = -D\left(\frac{\partial^2 u_{3b}}{\partial x_1^2} + \nu\frac{\partial^2 u_{3b}}{\partial x_2^2}\right) = f_{3b}(\partial\Omega_b), \quad D = \frac{Eh^2}{12(1 - \nu^2)} \tag{9.3}$$

intersecting force on the boundary

$$Q = -D\left(\frac{\partial^3 u_{3b}}{\partial x_2^3} + (2 - \nu)\frac{\partial^3 u_{3b}}{\partial x_1^2 \partial x_2}\right) = f_{4b}(\partial\Omega_b). \tag{9.4}$$

The correlations between the exertions on the stratified medium surface g_{kb}, $k = 1, 2, 3$ and between the dislocations u_k, $k = 1, 2, 3$ have the following form

$$u_3(x_1, x_2, x_3) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int K(\alpha_1, \alpha_2, x_3) \cdot G(\alpha_1, \alpha_2)e^{-i\langle\alpha, x\rangle}\, d\alpha_1\, d\alpha_2, \tag{9.5}$$

$$\langle\alpha, x\rangle = \alpha_1 x_1 + \alpha_2 x_2,$$

$$K(\alpha_1, \alpha_2, 0) = O(A^{-1}), \quad A = \sqrt{\alpha_1^2 + \alpha_2^2} \to \infty.$$

$K\left(\alpha_1, \alpha_2, x_3\right)$ – is an analytical function of two complex variables ,and particularly, meromorphic function, it's numerous examples are given in [10–13].

In case of two plates contacting longwise the axis let us give the parameters of the left an index , and of the right an index. Then the functional equation of the boundary problem for the left half plane can be represented in the following form

$$R_\lambda(-i\alpha_1, -i\alpha_2)U_{3\lambda} \equiv \varepsilon_{3\lambda}(\alpha_1^2 + \alpha_2^2)^2 U_{3\lambda} = -\int_{\partial\Omega_\lambda} \omega_\lambda + S_{3\lambda}(\alpha_1, \alpha_2), \tag{9.6}$$

$$S_{3\lambda}(\alpha_1, \alpha_2) = \mathbf{F}_2(\alpha_1, \alpha_2)(g_{3\lambda} + t_{3\lambda}).$$

Analogously for the right half plane

$$R_r(-i\alpha_1, -i\alpha_2)U_{3r} \equiv \varepsilon_{3r}(\alpha_1^2 + \alpha_2^2)^2 U_{3r} = -\int_{\partial\Omega_r} \omega_r + S_{3r}(\alpha_1, \alpha_2),$$

$$S_{3r}(\alpha_1 m \alpha_2) = \mathbf{F}_2(\alpha_1, \alpha_2)(g_{3r} + t_{3r}). \tag{9.7}$$

Here ω_b – is an external form participating in the representation and having the following form

$$\omega_b = \varepsilon_{3b} e^{i\langle\alpha,x\rangle} \left\{ -\left[\frac{\partial^3 u_{3b}}{\partial x_2^3} - i\alpha_2 \frac{\partial^2 u_{3b}}{\partial x_2^2} - \alpha_2^2 \frac{\partial u_{3b}}{\partial x_2} + i\alpha_2^3 u_{3b} + 2\frac{\partial^3 u_{3b}}{\partial x_1^2 \partial x_2} - 2i\alpha_2 \frac{\partial^2 u_{3b}}{\partial x_1^2} \right] \mathrm{d}x_1 + \left[\frac{\partial^3 u_{3b}}{\partial x_1^3} - i\alpha_1 \frac{\partial^2 u_{3b}}{\partial x_1^2} - \alpha_1^2 \frac{\partial u_{3b}}{\partial x_1} + i\alpha_1^3 u_{3b} \right] \mathrm{d}x_2 \right\}.$$

Having accepted the designations of works [1,2], having calculated residue Leray forms, including twofold forms, pseudodifferential equations of boundary problem taking into account the accepted designations we can represent for the left half plane in the following form

$$\mathbf{F}_1^{-1}(\xi_1^\lambda) \left\langle - \int_{\partial\Omega_\lambda} \left\{ i\alpha_{2-} D_\lambda^{-1} M_\lambda - D_\lambda^{-1} Q_\lambda - (\alpha_{2-}^2 + \nu_\lambda \alpha_1^2) \frac{\partial u_{3\lambda}}{\partial x_2} + i\alpha_{2-} \left[\alpha_{2-}^2 + (2-\nu_\lambda)\alpha_1^2 \right] u_{3\lambda} \right\} e^{i\alpha_1 x_1} \,\mathrm{d}x_1 + \varepsilon_{5\lambda} S_{3\lambda}(\alpha_1, \alpha_{2-}) \right\rangle = 0,$$

$$\alpha_{2-} = -i\sqrt{\alpha_1^2}, \quad \xi_1^\lambda \in \partial\Omega_\lambda,$$

$$\mathbf{F}_1^{-1}(\xi_1^\lambda) \left\langle - \int_{\partial\Omega_\lambda} \left\{ iD_\lambda^{-1} M_\lambda - 2\alpha_{2-} \frac{\partial u_{3\lambda}}{\partial x_2} + i\left[3\alpha_{2-}^2 + 2(2-\nu_\lambda)\alpha_1^2 \right] u_{3\lambda} \right\} e^{i\alpha_1 x_1} \,\mathrm{d}x_1 + \varepsilon_{5\lambda} S'_{3\lambda}(\alpha_1, \alpha_{2-}) \right\rangle = 0,$$

$$\xi_1^\lambda \in \partial\Omega_\lambda,$$

Accordingly for the right

$$\mathbf{F}_1^{-1}(\xi_1^r) \left\langle - \int_{\partial\Omega_r} \left\{ i\alpha_{2+} D_r^{-1} M_r - D_r^{-1} Q_r - (\alpha_{2+}^2 + \nu_r \alpha_1^2) \frac{\partial u_{3r}}{\partial x_2} + i\alpha_{2+} \left[\alpha_{2+}^2 + (2-\nu_r)\alpha_1^2 \right] u_{3r} \right\} e^{i\alpha_1 x_1} \,\mathrm{d}x_1 + \varepsilon_{5r} S_{3r}(\alpha_1, \alpha_{2+}) \right\rangle = 0,$$

$$\alpha_{2+} = i\sqrt{\alpha_1^2}, \quad \xi_1^r \in \partial\Omega_r,$$

$$\mathbf{F}_1^{-1}(\xi_1^r) \left\langle - \int_{\partial\Omega_r} \left\{ iD_r^{-1} M_r - 2\alpha_{2+} \frac{\partial u_{3r}}{\partial x_2} + i\left[3\alpha_{2+}^2 + 2(2-\nu_r)\alpha_1^2 \right] u_{3r} \right\} e^{i\alpha_1 x_1} \,\mathrm{d}x_1 + \varepsilon_{5r} S'_{3r}(\alpha_1, \alpha_{2+}) \right\rangle = 0,$$

$$\xi_1^r \in \partial\Omega_r.$$

The derivative is calculated with the parameter. Let us introduce the following notation system basing on (9.1)–(9.4)

$$\mathbf{Y}_\lambda = \{y_{1\lambda}, y_{2\lambda}\}, \quad \mathbf{Z}_\lambda = \{z_{1\lambda}, z_{2\lambda}\},$$

$$\mathbf{Y}_r = \{y_{1r}, y_{2r}\}, \quad \mathbf{Z}_r = \{z_{1r}, z_{2r}\},$$

$$\mathbf{F}_1 g = \mathbf{F}_1(\alpha_1) g, \quad \mathbf{F}_2 g = \mathbf{F}_2(\alpha_1, \alpha_2) g,$$

$$y_{1\lambda} = D_\lambda^{-1} \mathbf{F}_1 M_\lambda, \quad y_{2\lambda} = D_\lambda^{-1} \mathbf{F}_1 Q_\lambda,$$

$$y_{1r} = D_r^{-1} \mathbf{F}_1 M_r, \quad y_{2r} = D_r^{-1} \mathbf{F}_1 Q_r,$$

$$z_{1\lambda} = \mathbf{F}_1 \frac{\partial u_\lambda}{\partial x_2^\lambda}, \quad z_{2\lambda} = \mathbf{F}_1 u_\lambda,$$

$$z_{1r} = \mathbf{F}_1 \frac{\partial u_r}{\partial x_2^r}, \quad z_{2r} = \mathbf{F}_1 u_r,$$

$$\mathbf{K}_\lambda = \{k_{1\lambda}, k_{2\lambda}\}, \quad \mathbf{K}_r = \{k_{1r}, k_{2r}\},$$

$$k_{1\lambda} = \varepsilon_{5\lambda} \mathbf{F}_2(\alpha_1, \alpha_{21-})(g_\lambda + t_\lambda) = \varepsilon_{5\lambda} S_{3\lambda}(\alpha_1, \alpha_{2-}),$$

$$k_{2\lambda} = \varepsilon_{5\lambda} S'_{3\lambda}(\alpha_1, \alpha_{2-}),$$

$$k_{1r} = \varepsilon_{5r} \mathbf{F}_2(\alpha_1, \alpha_{21+})(g_r + t_r) = \varepsilon_{5r} S_{3r}(\alpha_1, \alpha_{2+}),$$

$$k_{2r} = \varepsilon_{5r} S'_{3r}(\alpha_1, \alpha_{2+}).$$

As a result the pseudodifferential equations for this case can be rewrote in the form of algebraic equation system

$$-i\alpha_{2-} y_{1\lambda} + y_{2\lambda} + (\alpha_{2-}^2 + \nu_\lambda \alpha_1^2) z_{1\lambda} - i\alpha_{2-} \left[\alpha_{2-}^2 + (2-\nu_\lambda)\alpha_1^2 \right] z_{2\lambda} + k_{1\lambda} = 0,$$

$$-iy_{1\lambda} + 2\alpha_{2-} z_{1\lambda} - i\left[3\alpha_{2-}^2 + 2(2-\nu_\lambda)\alpha_1^2 \right] z_{2\lambda} + k_{2\lambda} = 0,$$

$$- i\alpha_{2+}y_{1r} + y_{2r} + (\alpha_{2+}^2 + \nu_r\alpha_1^2)z_{1r} \\ - i\alpha_{2+}\left[\alpha_{2+}^2 + (2-\nu_r)\alpha_1^2\right] z_{2r} + k_{1r} = 0,$$

$$- iy_{1r} + 2\alpha_{2+}z_{1r} \\ - i\left[3\alpha_{2+}^2 + 2(2-\nu_r)\alpha_1^2\right] z_{2r} + k_{2r} = 0.$$

In the matrix form the system has the following form

$$\mathbf{A}_\lambda\mathbf{Y}_\lambda + \mathbf{B}_\lambda\mathbf{Z}_\lambda + \mathbf{K}_\lambda = 0,$$
$$\mathbf{A}_r\mathbf{Y}_r + \mathbf{B}_r\mathbf{Z}_r + \mathbf{K}_r = 0,$$

$$\mathbf{A}_{\boldsymbol\lambda} = \begin{Vmatrix} a_{11\lambda} & a_{12\lambda} \\ a_{21\lambda} & a_{22\lambda}\end{Vmatrix}, \quad \mathbf{B}_{\boldsymbol\lambda} = \begin{Vmatrix} b_{11\lambda} & b_{12\lambda} \\ b_{21\lambda} & b_{22\lambda}\end{Vmatrix},$$

$$\mathbf{A}_r = \begin{Vmatrix} a_{11r} & a_{12r} \\ a_{21r} & a_{22r}\end{Vmatrix}, \quad \mathbf{B}_r = \begin{Vmatrix} b_{11r} & b_{12r} \\ b_{21r} & b_{22r}\end{Vmatrix},$$

$$\begin{gathered} a_{11\lambda} = -i\alpha_{2-}, \quad a_{12\lambda} = 1, \quad a_{21\lambda} = -i, \\ a_{22\lambda} = 0, \quad b_{11\lambda} = (\alpha_{2-}^2 + \nu_\lambda\alpha_1^2), \\ b_{12\lambda} = -i\alpha_{2-}\left[\alpha_{2-}^2 + (2-\nu_\lambda)\alpha_1^2\right], \\ b_{21\lambda} = 2\alpha_{2-}, \\ b_{22\lambda} = -i\left[3\alpha_{2-}^2 + 2(2-\nu_\lambda)\alpha_1^2\right], \\ a_{11r} = -i\alpha_{2+}, \quad a_{12r} = 1, \quad a_{21r} = -i, \\ a_{22r} = 0, \quad b_{11r} = (\alpha_{2+}^2 + \nu_r\alpha_1^2), \\ b_{12r} = -i\alpha_{2+}\left[\alpha_{2+}^2 + (2-\nu_r)\alpha_1^2\right], \\ b_{21r} = 2\alpha_{2+}, \\ b_{22r} = -i\left[3\alpha_{2+}^2 + 2(2-\nu_r)\alpha_1^2\right]. \end{gathered} \tag{9.8}$$

Having solved these pseudodifferential equations for the chosen boundary problem and having inserted the found variables into the external forms in (9.6), (9.7), Fourier transform solution for plates can be represented for the left half plane $b = \lambda$ and for the right $b = r$ in a one-type form

$$U_{3b} = \left[R_b(-i\alpha_{1b}, -i\alpha_{2b})\right]^{-1} \\ \cdot \left\langle -\int\limits_{\partial\Omega_b} \omega_b + \varepsilon_{5b}\mathbf{F}_2(g_{3b} + t_{3b})\right\rangle.$$

The further utilizing of this representation for coupling with the substrate is described in [1,2] and applied below. Let us note that this investigation demands using different variants of integral factorization method [14,15].

2. The topological method of boundary problems resolution has an important advantage that is the coverage of all boundary problems types assumed by the considered boundary problem and also allows to investigate them in a one-type way. The latter allows comparing the resolutions of these problems visually, this is showed below. Let us consider some examples of cracked surfaces. In case of absence of defect exertion and dislocation of crack banks have to coincide. Let us study the case when the flaw represents free from bank exertion cracks, in other words. Then from the system (9.8) we found

$$\mathbf{Z}_\lambda = -\mathbf{B}_\lambda^{-1}\mathbf{K}_\lambda, \quad \mathbf{Z}_r = -\mathbf{B}_r^{-1}\mathbf{K}_r. \tag{9.9}$$

Let us consider the case when $\mathbf{Z}_\lambda = \mathbf{Z}_r$, $y_{1\lambda} = y_{1r} = 0$. Then the resolution comes from the correlations

$$\begin{gathered} \mathbf{Y}_{\lambda r} = -\mathbf{C}_{\lambda r}^{-1}(\mathbf{B}_\lambda^{-1}\mathbf{K}_\lambda - \mathbf{B}_r^{-1}\mathbf{K}_r), \\ \mathbf{C}_\lambda = \mathbf{B}_\lambda^{-1}\mathbf{A}_\lambda, \quad \mathbf{C}_r = \mathbf{B}_r^{-1}\mathbf{A}_r, \\ \mathbf{C}_{\lambda\mathbf{r}} = \begin{Vmatrix} c_{12\lambda} & -c_{12r} \\ c_{22\lambda} & -c_{22r}\end{Vmatrix}, \\ \mathbf{Y}_{\lambda r} = \{y_{2\lambda}, y_{2r}\}, \quad \mathbf{Y}_\lambda = \{0, y_{2\lambda}\}, \\ \mathbf{Y}_r = \{0, y_{2r}\}, \\ \mathbf{Z}_\lambda = -\mathbf{B}_\lambda^{-1}\mathbf{A}_\lambda\mathbf{Y}_\lambda - \mathbf{B}_\lambda^{-1}\mathbf{K}_\lambda. \end{gathered} \tag{9.10}$$

This case can be referred to presence of hidden flaw category unobserved by sight because dislocations and rotation angles of plates on the flaw are continued. Nevertheless, there exists the violation connectivity for the components of the stress, that is indicative of presence of errors (faults). Such examples of different types of errors, cracks, fractures can be continued. To understand the possible reason for destruction of the coating in the place of study, it's worthwhile to study the case of absence of the error and to define stressedly-deformed state in this case. Then should be accepted $\mathbf{Y}_\lambda = \mathbf{Y}_r$, $\mathbf{Z}_\lambda = \mathbf{Z}_r$. As a result we have from (9.8)

$$\mathbf{Y}_\lambda = (\mathbf{A}_\lambda^{-1}\mathbf{B}_\lambda - \mathbf{A}_r^{-1}\mathbf{B}_r)^{-1} \\ \cdot (\mathbf{A}_\lambda^{-1}\mathbf{K}_\lambda - \mathbf{A}_r^{-1}\mathbf{K}_r),$$

$$\mathbf{Z}_\lambda = (\mathbf{B}_\lambda^{-1}\mathbf{A}_\lambda - \mathbf{B}_\mathbf{r}^{-1}\mathbf{A}_r)^{-1} \\ \cdot (\mathbf{B}_\lambda^{-1}\mathbf{K}_\lambda - \mathbf{B}_r^{-1}\mathbf{K}_r). \tag{9.11}$$

Thus, the given examples show, that the topological method for the given type of boundary-value problems is very opportune. It allows the single-type research for the all-types problems both with and without errors. Having extracted all the pseudodifferential equations for each part of the limit and for each block, having included into them the corresponding boundary conditions, having solved the extracted from the pseudodifferential equations integral equations, we'll have from (9.6), (9.7) the representation of solutions in every block, that represents the plane in the form of

$$u_{3\lambda} = \mathbf{F}_2^{-1} \left[R_\lambda(-i\alpha_1, -i\alpha_2)\right]^{-1} \cdot \left\langle -\int\limits_{\partial\Omega_\lambda} \omega_\lambda + \varepsilon_{5\lambda}\mathbf{F}_2(g_\lambda + t_\lambda)\right\rangle,$$

$$u_{3r} = \mathbf{F}_2^{-1} \left[R_r(-i\alpha_1, -i\alpha_2)\right]^{-1} \cdot \left\langle -\int\limits_{\partial\Omega_r} \omega_r + \varepsilon_{5r}\mathbf{F}_2(g_r + t_r)\right\rangle. \quad (9.12)$$

3. Let's take the correlation (9.5), when in the form of

$$P_\lambda u_3(x_1, x_2, 0) + P_r u_3(x_1, x_2, 0) = F_2^{-1} K(\alpha_1, \alpha_2, 0) \cdot \left[G_\lambda(\alpha_1, \alpha_2) + G_r(\alpha_1, \alpha_2)\right], \quad (9.13)$$

$$G_\lambda(\alpha_1, \alpha_2) = F_2 P_\lambda g(x_1, x_2),$$
$$G_r(\alpha_1, \alpha_2) = F_2 P_r g(x_1, x_2).$$

Here $\mathbf{P}_\lambda$, $\mathbf{P}_r$ the projectors on the left and the right half-planes, which are the carriers of the corresponding panels. Inserting the correlation (9.12) into the left sides (9.13) and applying the Fourier transformations, we get the correlation in the form of

$$\left[R_\lambda(-i\alpha_1, -i\alpha_2)\right]^{-1} \cdot \left\langle -\int\limits_{\partial\Omega_\lambda} \omega_\lambda + \varepsilon_{5\lambda}(G_\lambda + T_\lambda)\right\rangle + \left[R_r(-i\alpha_1, -i\alpha_2)\right]^{-1} \cdot \left\langle -\int\limits_{\partial\Omega_r} \omega_r + \varepsilon_{5r}(G_r + T_r)\right\rangle$$
$$-K(\alpha_1, \alpha_2, 0)\left[G_\lambda(\alpha_1, \alpha_2) + G_r(\alpha_1, \alpha_2)\right] = 0,$$

$$T_\lambda = \mathbf{F}_2 t_\lambda(x_1, x_2), \quad T_r = \mathbf{F}_2 t_r(x_1, x_2).$$

The functions $G_\lambda(\alpha_1, \alpha_2)$, $G_r(\alpha_1, \alpha_2)$, which serve as the Fourier transformations of the functions, with the bearers in the half-planes, serve as the regular functions of operation factors α_2 with the fixed α_1 in the left and the right half-planes pro tanto. In this connection we can designate

$$G_\lambda(\alpha_1, \alpha_2) = G_-(\alpha_1, \alpha_2),$$
$$G_r(\alpha_1, \alpha_2) = G_+(\alpha_1, \alpha_2).$$

Inserting these designations into the previous correlation, we get to the Wiener–Hopf functional equation of the following form

$$MG_+ = G_- + V,$$
$$M = K_1 K_2^{-1}, \quad K_1 = R_r^{-1}\varepsilon_{5r} - K,$$
$$K_2 = K - R_\lambda^{-1}\varepsilon_{5\lambda}, \quad (9.14)$$

$$V = K_2^{-1}\left(R_\lambda^{-1}\int\limits_{\partial\Omega_\lambda} \omega_\lambda + R_r^{-1}\int\limits_{\partial\Omega_r} \omega_r - R_\lambda^{-1}\varepsilon_\lambda T_\lambda - R_r^{-1}\varepsilon_r T_r\right).$$

Solution of the functional equation (9.14) is not difficult. The methods of constructing of it's exact or approximate solutions we can find in works [10–15]. Taking into account, that with $\alpha_2 \to \pm\infty$ occurs the correlation $M \to \text{const}$, the solution may be written in the form of

$$G_+ = M_+^{-1}\left\{M_-^{-1}V\right\}^+,$$
$$G_- = -M_-\left\{M_-^{-1}V\right\}^-,$$
$$M = M_+ M_-,$$
$$M_-^{-1}V = \left\{M_-^{-1}V\right\}^+ + \left\{M_-^{-1}V\right\}^-.$$

Here the designation of work [11] is accepted.

The so-organized solutions have the following structure

$$G_+(\alpha_1, \alpha_2) = C_{1+}(\alpha_1, \alpha_2)G_+(\alpha_1, \alpha_{2+}) + C_{2+}(\alpha_1, \alpha_2)G_-(\alpha_1, \alpha_{2-}) + C_{3+}(\alpha_1, \alpha_2)G'_+(\alpha_1, \alpha_{2+}) + C_{4+}(\alpha_1, \alpha_2)G'_-(\alpha_1, \alpha_{2-}) + C_{5+}(\alpha_1, \alpha_2), \quad (9.15)$$

$$G_-(\alpha_1, \alpha_2) = C_{1-}(\alpha_1, \alpha_2)G_+(\alpha_1, \alpha_{2+}) + C_{2-}(\alpha_1, \alpha_2)G_-(\alpha_1, \alpha_{2-}) + C_{1-}(\alpha_1, \alpha_2)G'_+(\alpha_1, \alpha_{2+}) + C_{2-}(\alpha_1, \alpha_2)G'_-(\alpha_1, \alpha_{2-}) + C_{3-}(\alpha_1, \alpha_2).$$

Let's differentiate the first and the second equation according to α_2.

Here the functions

$$C_{n+}(\alpha_1, \alpha_2),\ C_{n-}(\alpha_1, \alpha_2),\ n = 1, 2, 3$$

are known, and the functions

$$G_{+}(\alpha_1, \alpha_{2-}),\ G_{-}(\alpha_1, \alpha_{2+}),$$

$$G'_{+}(\alpha_1, \alpha_{2+}),\ G'_{-}(\alpha_1, \alpha_{2-})$$

should be defined. For their definition let's take in the first and differentiated equation $\alpha_2 = \alpha_{2+}$, and in the second and differentiated equation $\alpha_2 = \alpha_{2-}$. We get the algebraic system fot the definition of all of the above-listed indeterminates, which we should solve to get the decision functions. The entry of the computed solutions into the correlations (9.9)–(9.11), depending on the stated boundary-value problem, then using the correlations (9.15), (9.12) gives the opportunity to determine completely stressedly-deformed condition of coating with any of the concerned errors and without them.

10. The Model the Starting Earthquake

In the work is represented obviously for the first time the model of one type of earthquakes beginning from the preparation to the accomplishment of the event. The model based fully on the laws of physics and mechanics may reveal the new type of faulting earthquake called the starting one, as it precedes to strong crustal earthquakes, connected with the lithosphere plates' interaction [1]. As lithospheric plates we take the Kirchhoff plates on elastic half-space moving to each other till they approach. The earthquake is defined by drastic increase of stress concentration in a specified area in comparison with a normal condition. The mining allows evaluating with the aid of specific equipment the location, time and intensity of this type of earthquakes. The patterns of this earthquake are revealed.

1. A great number of works made by scientists from our as well as other countries are dedicated to the earthquakes research. The essential results that defined the direction of researches on various steps are made in these works. There the processes of earthquakes running and their impact on the environment are studied in details. However, the problem of their antecedents, i.e. prospective earthquake locations, time and intensity of events could not be solved. The development of new means of geophysical information obtaining, of new types of equipment for the deep Earth structure exploration and also of computing tools and mathematical methods allow to indicate and solve more difficult problems in seismology than earlier. In the earthquakes research two different approaches are distinguished, i.e. the approach of observational and active seismology. The first one implies the observation consisting in using lots of observational devices followed by the analysis and revealing the parameters of forthcoming earthquakes. The active one is based on seismic activity theories building, using as its base the observation data and laws of physics, and on application of addressed means of impact made by powerful vibroseismic sources on specified areas of surface for revealing the antecedents' parameters. Mr. Keilis-Borok [9], academician, wrote about the first approach the following: "For a long time the works on the earthquakes forecasting were oriented generally on the observation system expanding. The earthquake in California showed us that it is not enough. It occurred in the centre of the most powerful observation system in the world, with thousands of sensors, telemetry and total computerization". The second approach was described in details in a collective monograph under the guidance of academician, Mr. Alekseev [10]. He also assumes the building of deep theories of earthquakes based on physico-mechanical laws which stimulated the directed studying of various types of these events conducted, for example, in [11–13]. The most profound considerations for earthquakes forecast forming were proposed by the former directors of the Institute of Physics of the Earth of the Academy of Sciences of the USSR, academicians, Mr. Gamburtsev and Mr. Sadovsky. Their ideas, in authors' opinion, are the manual in studying antecedents on the modern stage. Mr. Gamburtsev had the following point of view [6]: "The investigation of methods of earthquakes forecast should be directed first of all to the searching of mechanical earthquakes antecedents. Such searching can be successful only if they will be based on profound studying of all the details of the mechanism of quick and slow movements of lithospheric blocks in the areas of seismic activity. Mr. Sadovsky [7] states that it is impossible to predict earthquakes taking as its base only the layered structure of lithosphere. It is necessary to take into account the actually existing bock models.

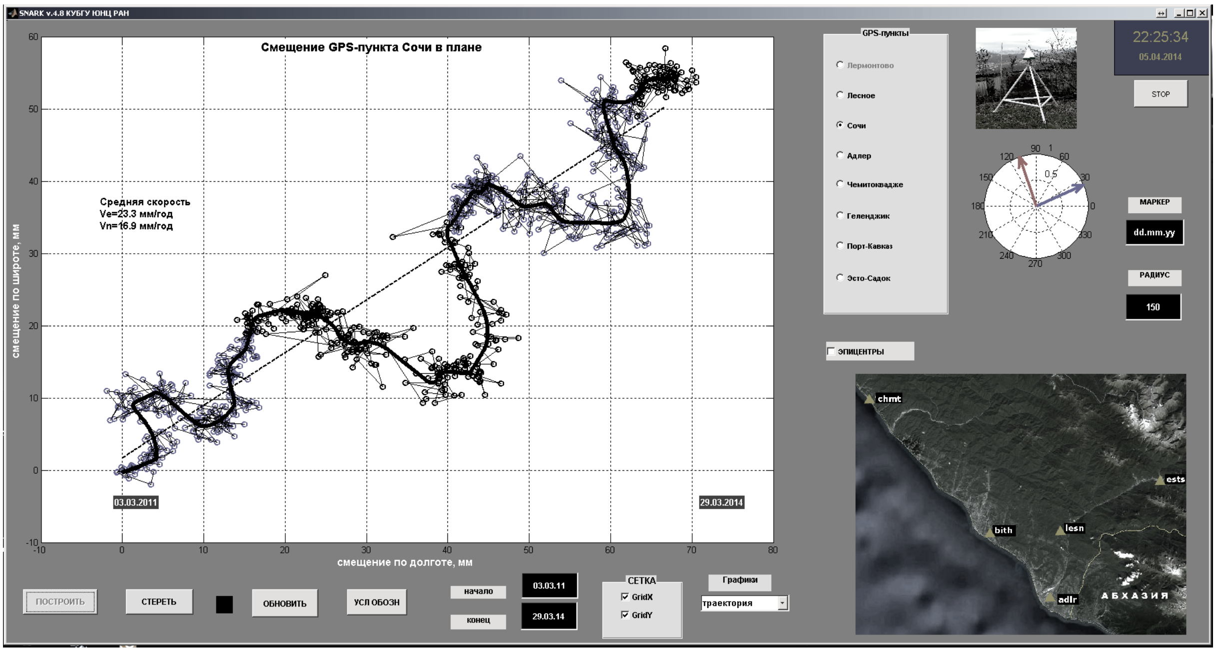

Figure 1. The slow movement of the mounting point of the GPS receiver in Sochi

2. The conducted research is based on the theory of concealed defects worked out in [14] and other authors' publications. Let us use the schemes made for describing concealed defects in environments with coverings [14] assuming that the coverings are half-plates with parallel boarders with the distance of 2θ between them, placed on some linear deformed foundation. Lithospheric plates are shaped by the Kirchhoff plates. Let us assume that the space between the plates of different types is hollow and on the butt-ends the outside powers are acting, which are directed according to the rule of outward vectors in a local system of coordinates $x_1x_2x_3$ with the beginning in the plane x_1x_2 coinciding with the middle plane of the plate, with the axis ox_3 directed upward normally to the plate, with the axis ox_1 directed along the tangent to the fault zone edge, with the axis ox_2 normally to its edge. The area occupied by the left plate is marked by λ and described by the correlations $|x_1| \leqslant \infty$, $x_2 \leqslant -\theta$, and the area occupied by the right one is marked by the index r and coordinates $|x_1| \leqslant \infty$, $\theta \leqslant x_2$. We will proceed from the fact that the lithospheric plates move extremely slow. On the Fig. 1 we can see the speed of point indicating the location of high-accuracy GPS receiver. The speed is about tens of millimeters annually, that is why the boarder task can be examined in static variant.

Kirchhoff's equation for fragments of b cover, $b = \lambda, r$, which occupy interval Ω_b with borders $\partial\Omega_b$ at vertical static exertion impacts t_{3b} from above and g_{3b} from below, looks like

$$\begin{aligned}\mathbf{R}_b(\partial x_1, \partial x_2)u_{3b} + \varepsilon_{53b}(t_{3b} - g_{3b}) \\ \equiv \left(\frac{\partial^4}{\partial x_1^4} + 2\frac{\partial^2}{\partial x_1^2}\frac{\partial^2}{\partial x_2^2} + \frac{\partial^4}{\partial x_2^4}\right) u_{3b} \\ + \varepsilon_{53b}(t_{3b} - g_{3b}) = 0,\end{aligned}$$

$$\begin{aligned}\mathbf{R}_b(-i\alpha_1, -i\alpha_2)U_{3b} \equiv R_b(-i\alpha_1, -i\alpha_2)U_{3b} \\ \equiv (\alpha_1^2 + \alpha_2^2)^2 U_{3b},\end{aligned}$$

$$U_{3b} = \mathbf{F}_2 u_{3b}, \quad G_{3b} = \mathbf{F}_2 g_{3b},$$

$$T_{3b} = \mathbf{F}_2 t_{3b}, \quad b = \lambda, r,$$

$$M_b = -D_{b1}\left(\frac{\partial^2 u_{3b}}{\partial x_2^2} + \nu_b \frac{\partial^2 u_{3b}}{\partial x_1^2}\right),$$

$$D_{b1} = \frac{D_b}{H^2}, \quad D_{b2} = \frac{D_b}{H^3},$$

$$\begin{aligned}Q_b = -D_{b2}\left(\frac{\partial^3 u_{3b}}{\partial x_2^3} + (2 - \nu_b)\frac{\partial^3 u_{3b}}{\partial x_1^2 \partial x_2}\right) \\ = f_{4b}(\partial\Omega_b),\end{aligned}$$

$$u_{3b} = f_{1b}(\partial\Omega_b), \quad \frac{\partial u_{3b}}{H\partial x_2} = f_{2b}(\partial\Omega_b),$$

$$D_b = \frac{E_b h_b^3}{12(1 - \nu_b^2)}, \quad \varepsilon_{53b} = \frac{(1 - \nu_b^2)12H^4}{E_b h_b^3},$$

$$\varepsilon_6^{-1} = \frac{(1-\nu)H}{\mu}.$$

The connection between boundary stress and surface displacement of elastic earth, where plates are situated looks like

$$u_{3m}(x_1,x_2) = \varepsilon_6^{-1}\sum_{n=1}^{2}\iint_{\Omega_n} k(x_1-\xi_1, x_2-\xi_2) \cdot g_{3n}(\xi_1,\xi_2)\,\mathrm{d}\xi_1\,\mathrm{d}\xi_2,$$

$$x_1, x_2 \in \Omega_m, \quad m = \lambda, r, \theta,$$

$$\Omega_\lambda(|x_1| \leqslant \infty;\ x_2 \leqslant -\theta),$$

$$\Omega_r(|x_1| \leqslant \infty;\ \theta \leqslant x_2),$$

$$\Omega_\theta(|x_1| \leqslant \infty;\ -\theta \leqslant x_2 \leqslant \theta), \quad n = \lambda, r.$$

M_b and Q_b are bending moment and sharing force in the system of axis x_1ox_2; h_b is plate thicknesses, H is dimensional parameter of base material, for example, layer thickness. The designation is borrowed from [14]. $\mathbf{F}_2 \equiv \mathbf{F}_2(\alpha_1,\alpha_2)$, and $\mathbf{F}_1 \equiv \mathbf{F}_1(\alpha_1)$ are two-dimensional and one-dimensional Fourier-transform operators, apparently. Functional equations of boundary-value problems can be represented as [14]

$$R_b(-i\alpha_1,-i\alpha_2)U_{3b} \equiv (\alpha_1^2+\alpha_2^2)^2 U_{3b} = -\int_{\partial\Omega_b} \omega_b - \varepsilon_{53b}S_{3b}(\alpha_1,\alpha_2), \quad (10.1)$$

$$S_{3b}(\alpha_1,\alpha_2) = \mathbf{F}_2(\alpha_1,\alpha_2)(t_{3b}-g_{3b}), \quad b = \lambda, r.$$

Here we can see that ω_b are exterior forms, participating in performance and looking like

$$\omega_b = e^{i\langle\alpha,x\rangle}\Bigg\{-\Big[\frac{\partial^3 u_{3b}}{\partial x_2^3} - i\alpha_2\frac{\partial^2 u_{3b}}{\partial x_2^2} - \alpha_2^2\frac{\partial u_{3b}}{\partial x_2} + i\alpha_2^3 u_{3b} + 2\frac{\partial^3 u_{3b}}{\partial x_1^2\partial x_2} - 2i\alpha_2\frac{\partial^2 u_{3b}}{\partial x_1^2}\Big]\,\mathrm{d}x_1 + \Big[\frac{\partial^3 u_{3b}}{\partial x_1^3} - i\alpha_1\frac{\partial^2 u_{3b}}{\partial x_1^2} - \alpha_1^2\frac{\partial u_{3b}}{\partial x_1} + i\alpha_1^3 u_{3b}\Big]\,\mathrm{d}x_2\Bigg\},$$

$$b = \lambda, r.$$

Calculating Leray's residue forms, including two-fold and pseudodifferential equations of boundary-value problems and taking into account agreed notations we can introduce for plates $b = \lambda, r$ as

$$\mathbf{F}_1^{-1}(\xi_1^\lambda)\Bigg\langle -\int_{\partial\Omega_\lambda}\Big\{i\alpha_{2-}D_{\lambda 1}^{-1}M_\lambda - D_{\lambda 2}^{-1}Q_\lambda - (\alpha_{2-}^2+\nu_\lambda\alpha_1^2)\frac{\partial u_{3\lambda}}{\partial x_2} + i\alpha_{2-}\left[\alpha_{2-}^2+(2-\nu_\lambda)\alpha_1^2\right]u_{3\lambda}\Big\}e^{i\alpha_1 x_1}\,\mathrm{d}x_1 + \varepsilon_{53\lambda}S_{3\lambda}(\alpha_1,\alpha_{2-})\Bigg\rangle = 0, \quad (10.2)$$

$$\alpha_{2-} = -i\sqrt{\alpha_1^2}, \quad \xi_1^\lambda \in \partial\Omega_\lambda,$$

$$\mathbf{F}_1^{-1}(\xi_1^\lambda)\Bigg\langle -\int_{\partial\Omega_\lambda}\Big\{iD_{\lambda 1}^{-1}M_\lambda - 2\alpha_{2-}\frac{\partial u_{3\lambda}}{\partial x_2} + i\left[3\alpha_{2-}^2+(2-\nu_\lambda)\alpha_1^2\right]u_{3\lambda}\Big\}e^{i\alpha_1 x_1}\,\mathrm{d}x_1 + \varepsilon_{53\lambda}S'_{3\lambda}(\alpha_1,\alpha_{2-})\Bigg\rangle = 0,$$

$$\xi_1^\lambda \in \partial\Omega_\lambda, \quad \partial\Omega_\lambda = \{-\infty \leqslant x_1 \leqslant \infty, x_2 = -\theta\}.$$

And consequently for the right plate

$$\mathbf{F}_1^{-1}(\xi_1^r)\Bigg\langle -\int_{\partial\Omega_r}\Big\{i\alpha_{2+}D_{r1}^{-1}M_r - D_{r2}^{-1}Q_r - (\alpha_{2+}^2+\nu_r\alpha_1^2)\frac{\partial u_{3r}}{\partial x_2} + i\alpha_{2+}\left[\alpha_{2+}^2+(2-\nu_r)\alpha_1^2\right]u_{3r}\Big\}e^{i\alpha_1 x_1}\,\mathrm{d}x_1 + \varepsilon_{53r}S_{3r}(\alpha_1,\alpha_{2+})\Bigg\rangle = 0, \quad (10.3)$$

$$\alpha_{2+} = i\sqrt{\alpha_1^2}, \quad \xi_1^r \in \partial\Omega_r,$$

$$\mathbf{F}_1^{-1}(\xi_1^r)\Bigg\langle -\int_{\partial\Omega_r}\Big\{iD_{r1}^{-1}M_r - 2\alpha_{2+}\frac{\partial u_{3r}}{\partial x_2} + i\left[3\alpha_{2+}^2+(2-\nu_r)\alpha_1^2\right]u_{3r}\Big\}e^{i\alpha_1 x_1}\,\mathrm{d}x_1 + \varepsilon_{53r}S'_{3r}(\alpha_1,\alpha_{2+})\Bigg\rangle = 0,$$

$$\xi_1^r \in \partial\Omega_r, \quad \partial\Omega_r = \{-\infty \leqslant x_1 \leqslant \infty,\ x_2 = \theta\}.$$

The derivative is calculated from parameter α_2. Let's introduce the next system of notations on basis of (10.2) and (10.3)

$$\mathbf{Y}_\lambda = \{y_{1\lambda}, y_{2\lambda}\}, \quad \mathbf{Z}_\lambda = \{z_{1\lambda}, z_{2\lambda}\},$$

$$\mathbf{Y}_r = \{y_{1r}, y_{2r}\}, \quad \mathbf{Z}_r = \{z_{1r}, z_{2r}\},$$

$$\mathbf{F}_1 g = \mathbf{F}_1(\alpha_1)g, \quad \mathbf{F}_2 g = \mathbf{F}_2(\alpha_1, \alpha_2)g,$$

$$y_{1\lambda} = D_\lambda^{-1}\mathbf{F}_1 M_\lambda, \quad y_{2\lambda} = D_\lambda^{-1}\mathbf{F}_1 Q_\lambda,$$

$$y_{1r} = D_r^{-1}\mathbf{F}_1 M_r, \quad y_{2r} = D_r^{-1}\mathbf{F}_1 Q_r,$$

$$z_{1\lambda} = \mathbf{F}_1 \frac{\partial u_\lambda}{\partial x_2^\lambda}, \quad z_{2\lambda} = \mathbf{F}_1 u_\lambda,$$

$$z_{1r} = \mathbf{F}_1 \frac{\partial u_r}{\partial x_2^r}, \quad z_{2r} = \mathbf{F}_1 u_r,$$

$$\mathbf{K}_\lambda = \{k_{1\lambda}, k_{2\lambda}\}, \quad \mathbf{K}_r = \{k_{1r}, k_{2r}\},$$

$$\begin{aligned} k_{1\lambda} &= \varepsilon_{53\lambda}\mathbf{F}_2(\alpha_1, \alpha_{2-})(t_\lambda - g_\lambda) \\ &= \varepsilon_{53\lambda} S_{3\lambda}(\alpha_1, \alpha_{2-}), \end{aligned}$$

$$k_{2\lambda} = \varepsilon_{53\lambda} S'_{3\lambda}(\alpha_1, \alpha_{2-}),$$

$$\begin{aligned} k_{1r} &= \varepsilon_{53r}\mathbf{F}_2(\alpha_1, \alpha_{2+})(t_\lambda - g_\lambda) \\ &= \varepsilon_{53r} S_{3r}(\alpha_1, \alpha_{2+}), \end{aligned}$$

$$k_{2r} = \varepsilon_{53r} S'_{3r}(\alpha_1, \alpha_{2+}).$$

As the result pseudodifferential equations for this case can be rewritten as a system of algebraic equations

$$\begin{aligned} &- i\alpha_{2-} y_{1\lambda} + y_{2\lambda} + (\alpha_{2-}^2 + \nu_\lambda \alpha_1^2) z_{1\lambda} \\ &\quad - i\alpha_{2-}\left[\alpha_{2-}^2 + (2 - \nu_\lambda)\alpha_1^2\right] z_{2\lambda} + k_{1\lambda} = 0, \end{aligned}$$

$$\begin{aligned} &- i y_{1\lambda} + 2\alpha_{2-} z_{1\lambda} \\ &\quad - i\left[3\alpha_{2-}^2 + (2 - \nu_\lambda)\alpha_1^2\right] z_{2\lambda} + k_{2\lambda} = 0, \end{aligned}$$

$$\begin{aligned} &- i\alpha_{2+} y_{1r} + y_{2r} + (\alpha_{2+}^2 + \nu_r \alpha_1^2) z_{1r} \\ &\quad - i\alpha_{2+}\left[\alpha_{2+}^2 + (2 - \nu_r)\alpha_1^2\right] z_{2r} + k_{1r} = 0, \end{aligned}$$

$$\begin{aligned} &- i y_{1r} + 2\alpha_{2+} z_{1r} \\ &\quad - i\left[3\alpha_{2+}^2 + (2 - \nu_r)\alpha_1^2\right] z_{2r} + k_{2r} = 0. \end{aligned}$$

In matrix form the system is given by

$$\mathbf{A}_\lambda \mathbf{Y}_\lambda + \mathbf{B}_\lambda \mathbf{Z}_\lambda + \mathbf{K}_\lambda = 0,$$

$$\mathbf{A}_r \mathbf{Y}_r + \mathbf{B}_r \mathbf{Z}_r + \mathbf{K}_r = 0.$$

For the sake of simplicity, let's examine the case when bonding moment and sharing force are equal to zero, then we get $\mathbf{Y}_\lambda = 0$, $\mathbf{Y}_r = 0$. The solutions of resulting equations are easy to find

$$\mathbf{Z}_\lambda = -\mathbf{B}_\lambda^{-1}\mathbf{K}_\lambda, \quad \mathbf{Z}_r = -\mathbf{B}_r^{-1}\mathbf{K}_r,$$

$$(-1+\nu_\lambda)\alpha_1^2 z_{1\lambda} - i\alpha_{2-}\left[(1-\nu_\lambda)\alpha_1^2\right] z_{2\lambda} = -k_{1\lambda},$$

$$2\alpha_{2-} z_{1\lambda} + i\left[(1+\nu_\lambda)\alpha_1^2\right] z_{2\lambda} = -k_{2\lambda}$$

$$(-1+\nu_r)\alpha_1^2 z_{1r} - i\alpha_{2+}\left[(1-\nu_r)\alpha_1^2\right] z_{2r} = -k_{1r},$$

$$2\alpha_{2+} z_{1r} + i\left[(1+\nu_r)\alpha_1^2\right] z_{2r} = -k_{2r},$$

$$\Delta_{\lambda 0} = -i(1-\nu_\lambda)(3+\nu_\lambda)\alpha_1^4,$$

$$\Delta_{r0} = -i(1-\nu_r)(3+\nu_r)\alpha_1^4,$$

$$z_{1\lambda} = \frac{i\alpha_1^2\left[-(1+\nu_\lambda)k_{1\lambda} - (1-\nu_\lambda)k_{2\lambda}\alpha_{2-}\right]}{\Delta_{\lambda 0}},$$

$$z_{2\lambda} = \frac{2\alpha_{2-}k_{1\lambda} + (1-\nu_\lambda)\alpha_1^2 k_{2\lambda}}{\Delta_{\lambda 0}},$$

$$z_{1r} = \frac{i\alpha_1^2\left[-(1+\nu_r)k_{1r} - (1-\nu_r)k_{2r}\alpha_{2-}\right]}{\Delta_{r0}},$$

$$z_{2r} = \frac{2\alpha_{2-}k_{1r} + (1-\nu_r)\alpha_1^2 k_{2r}}{\Delta_{r0}}.$$

After applying of found correlations in formulas for exterior forms in (10.2) and (10.3) we will have two equations for $\theta > 0$, and $\theta = 0$.

$$G_{3r} = G^+, \quad G_{3\lambda} = G^-,$$

$$\begin{aligned} &\left[\varepsilon_{53r}(\alpha_1^2 + \alpha_2^2)^{-2} + \varepsilon_6^{-1} K_1(\alpha_1, \alpha_2)\right] \\ &\qquad \cdot G^+(\alpha_1, \alpha_2) \\ &= -\left[\varepsilon_{53\lambda}(\alpha_1^2 + \alpha_2^2)^{-2} + \varepsilon_6^{-1} K_1(\alpha_1, \alpha_2)\right] \\ &\qquad \cdot G^-(\alpha_1, \alpha_2) + U_{3\theta}(\alpha_1, \alpha_2) \\ &+ (\alpha_1^2 + \alpha_2^2)^{-2}\big[A_\lambda k_{1\lambda} + B_\lambda k_{2\lambda} + A_r k_{1r} + B_r k_{2r} \\ &\qquad + \varepsilon_{53\lambda} T^+(\alpha_1, \alpha_2) + \varepsilon_{53r} T^-(\alpha_1, \alpha_2)\big], \end{aligned}$$

$$\theta > 0,$$

$$U_{3\theta}(\alpha_1, \alpha_2) = \int\limits_{-\infty}^{\infty}\int\limits_{-\theta}^{\theta} u_3(x_1, x_2) e^{i\langle \alpha, x\rangle}\, \mathrm{d}x_1\, \mathrm{d}x_2,$$

$$
\begin{gathered}
\left[\varepsilon_{53r}(\alpha_1^2+\alpha_2^2)^{-2}+\varepsilon_6^{-1}K_1(\alpha_1,\alpha_2)\right] \\
\cdot G^+(\alpha_1,\alpha_2) \\
=-\left[\varepsilon_{53\lambda}(\alpha_1^2+\alpha_2^2)^{-2}+\varepsilon_6^{-1}K_1(\alpha_1,\alpha_2)\right] \\
\cdot G^-(\alpha_1,\alpha_2) \\
+(\alpha_1^2+\alpha_2^2)^{-2}\left[A_\lambda k_{1\lambda}+B_\lambda k_{2\lambda}+A_r k_{1r}+B_r k_{2r}+\right. \\
\left.+\varepsilon_{53\lambda}T^+(\alpha_1,\alpha_2)+\varepsilon_{53r}T^-(\alpha_1,\alpha_2)\right], \\
\theta=0.
\end{gathered}
$$

As $\theta \to 0$, i.e. when the plates are approaching, the first equation incessantly passes into the second. We got two different Wiener-Hopf equations. The first is extended of variationals of Wiener–Hopf equation associated with the presence of function $U_{3\theta}(\alpha_1,\alpha_2)$. It is solving by presented in [15] converse of system of two integrational equations of second kind with quite continuous operators as

$$
X^+ - \left\{-\frac{M_1^+}{M_2^-}Y^- e^{-i2\alpha_2\theta}\right\}^+ = \left\{\frac{1}{M_2^-}\varphi e^{-i\alpha_2\theta}\right\}^+,
$$

$$
Y^- + \left\{\frac{M_2^-}{M_1^+}X^+ e^{i2\alpha_2\theta}\right\}^- = \left\{\frac{1}{M_1^+}\varphi e^{i\alpha_2\theta}\right\}^-,
$$

$$
M_1 = M_1^+ M_1^-, \quad M_2 = M_2^+ M_2^-,
$$

$$
M_2^+ G^+ = X^+, \quad M_1^- G^- = Y^-.
$$

The designations of work are accepted here [15].

In the process of solution of functional equation we have to designate functionals $S_{3b}(\alpha_1,\alpha_{2\pm})$, $b=\lambda,r$, from some system of equations [14].

3. While researching the solution of the first equation it is proved that for $\theta>0$. the next internals of contact stresses between plates and layers take place.

$$
g_{3\lambda}(x_1,x_2)=\sigma_{1\lambda}(x_1,x_2)(-x_2-\theta)^{-1/2}, \quad x_2<-\theta,
$$

$$
g_{3r}(x_1,x_2)=\sigma_{1r}(x_1,x_2)(x_2-\theta)^{-1/2}, \quad x_2>\theta.
$$

Here $\sigma_{1b}(x_1,x_2)$, $b=\lambda,r$, are incessant at the both coordinates of function for sufficiently smooth t_{3b}, $b=\lambda,r$ [15].

The conversion of the second equation at $\theta=0$ is forming by traditional method of Wiener–Hopf [15] and leads to the next internals of solutions at $x_2\to 0$

$$
\begin{aligned}
g_{3\lambda}(x_1,x_2) &\to \sigma_{2\lambda}(x_1,x_2)x_2^{-1} \\
&\quad +\sigma_{3\lambda}(x_1,x_2)\ln|x_2|, \\
g_{3r}(x_1,x_2) &\to \sigma_{2r}(x_1,x_2)x_2^{-1} \\
&\quad +\sigma_{3r}(x_1,x_2)\ln|x_2|.
\end{aligned} \tag{10.4}
$$

The functions $\sigma_{nb}(x_1,x_2)$, $b=\lambda,r$; $n=2,3$ are incessant at the both parameters.

Proceeding from the position that correlations (10.4) prove that there is detection of the new type of earthquake, let us describe its common factors.

1) At $\theta>0$ the plates act upon layer as simple stamps with right angles at borders [15]. If the edges break, then the facilities at edges disappear. This can induce a slight earthquake.

2) When the plates approached $\theta=0$, but did not combine, preserving specified boundary conditions on edges, and Newton's third law hasn't been there yet, then the irregularities between them occur x_2^{-1} and $\ln|x_2|$. For mechanics this is absolutely clear situation, described in many works and frequent, for example, in studies of solidity of metal angle, welded onto foundation. For some values of angular solution of angle there is the growth of exertion factors in vicinity of an angle. It indicates fast decomposition of combination under external influences, when the facility appears as non-totalize. When the plates did not combine until the accomplishment of Newton's third law, they can vertically slide with edges against one another and under other external influences upon them, like scissors, they can vertically rip open the layer or break. Such example of behavior of plates is well-known among fishers: when the ice is thin, it's better not to come closer, or you can fall through ice.

3) The decomposition happens as doublet, there are two facilities of exertion. G.A. Gandurtsev and other seismologists concluded from analysis of the first wave arrival about reiterated and different-type decomposition in earthquake focus. It is also possible that the large facility is a shock, and the small one is aftershock.

4) Academicians B.B. Golitsin, A. Gandurtsev and other scientists spoke up about the possibility of boundary adhesion of lithosphere plates as the result of breaches when they approach and about the possibility of the next layers, which lead to earthquakes. In our case at the same features of lithosphere plates and

Figure 2. Vibroseis source Failing Y-3000

matching of edges, the equation of displacement and exertions in plates contact zone, i.e. accomplishment of Newton's third law, leads to disappearing of facilities, and two plates transfer into one surface.

5) In order to use this result for practical purposes, for certification of condition of fractured zones we can apply heavy, weighing 30 tons seismic vibrator (Fig. 2), which are producing in USA, for example, Y-3000, by Failing company, and then supply such seismic hybrids with information handling projects, which will allow to realize monitoring of location of plates and to calculate places and time of occurrence. The simplest scalar interaction type of lithosphere plates is analyzed in the work. More complicated vector cases demand using of solution of Hilbert–Wiener problem, which was recently published by authors.

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