## Механика

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# ELECTRO-MAGNETO-ELASTIC SURFACE WAVES EXISTENCE AND PROPAGATION IN A PIEZOELECTRIC LAYERED MEDIUM IN THE PRESENCE OF AN ELECTRIC SCREEN

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*Abstract.* The problems of existence and behavior of electro-magneto-elastic surface waves in a structure consisting of a piezoelectric substrate, a piezoelectric layer and an adjoining dielectric medium on the top in the presence of an electric (or magnetic) screen are considered. The dispersion equation is derived and investigated.

*Keywords:* electro-magneto-elastic wave, layered piezoelectric structure, dynamic Maxwell's equations.

### Introduction

The proposed paper is an investigation of existence and propagation of electro-magneto-elastic surface waves in piezoelectric layered structures in the presence of an electric (or magnetic) screen when the equations of motion are considered along with complete set of Maxwell's equations for electro-magnetic fields.

Piezoelectric effect in piezoelectric crystals and ceramics leads to elastic and electromagnetic excitations coupling and as a result there appear various specific effects in a piezoelectric media. Recently conducted research for detection and study of such kind of effects considers the wave process as interconnected process of elastic and electromagnetic excitations propagation. Here are some examples: the existence of shear electro-elastic surface Bluestein–Gulyaev waves on the surface of the elastic piezoelectric half space, existence of surface waves along with refracted and reflected acoustic waves, the possibility of refraction and reflection without varying the polarization, existence of gap waves in the clearance between the piezoelectric layers and other effects.

It is known that the surface waves can be applied in various technical devices in particular in devices for storing, treaty and transferring the information. Consequently the investigation of the electro-magneto-elastic surface wave behavior in a piezoelectric media has an important theoretical and practical significance [1–21].

These investigations were started by Bluestein [4] and Gulyaev [5] and have been carried out since. Many of these investigations mainly have been done in quasi-static approximation when the wave character of electromagnetic field is not taken into account, and the electric and magnetic fields are not coupled, only the mechanical field is considered dynamic. Therefore the quasi-static approximation does not determine the electro-magnetic field excited by the mechanical deformation. Hence it is not possible to calculate the power of the electromagnetic energy radiated from a vibrating piezoelectric device, but it is relevant in acoustic delay lines, wireless acoustic wave sensors in which acoustic waves produce electromagnetic waves or vice versa. These and similar issues raising the necessity to investigate the wave

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process in piezoelectric in a dynamic settings when the equations of motion are considered along with dynamic Maxwell's equations for the electro-magnetic field [2, 6].

The dynamic theory of piezoelectromagnetism was formulated and investigated by R.D. Mindlin [1], I.S. Yang [2], S. Li [3], and others. Many researches are devoted to a surface shear wave propagation in layered structures consisting of a piezoelectric substrate and a piezoelectric [4–6, 11–13, 15, 17], conducting [12] or dielectric layer [19].

Investigation of surface waves in multilayered structures consisting of more than one layer was also a research topic of many papers. The present paper considers the propagation of the electro-magneto-elastic surface waves in a layered structure consisting of a piezoelectric half-space, an adjacent piezoelectric layer, the second layer is dielectric (or vacuum) and in the top of the structure there is an electric (or magnetic) screen. The expressions representing the wave fields and the dispersion equations are obtained and analyzed. The effect of the screen on the propagation behavior of the electro-magneto-elastic waves have been studied also.

## 1. Statement of the problem

We consider a layered structure consisting of an elastic piezoelectric layer and a piezoelectric half-space. Both the substrate and the layer belong to the crystal class 6 mm or 4 mm (Fig. 1). The $Ox_3$ axis is directed along the main crystal axis of symmetry ($L_6$ or $L_4$) of the piezoelectric substrate and the layer. The $Ox_2$ axis points down into the substrate. The layer with the thickness $h_2$ is rigidly linked to the substrate. The electric or magnetic screen is located at a distance $h_3$ from the layer. The domain $-(h_2+h_3) < x_2 < -h_2$ is assumed to be either a dielectric medium (or vacuum) without an acoustic contact with the piezoelectric layer. The layer surfaces $x_2 = 0$, $x_2 = -h_2$ are electrically open and the surface $x_2 = -h_2$ is free of external forces (mechanically free). Here after superscripts 1, 2, 3 indicate that the value belongs to the substrate, layer and the dielectric medium respectively. $c_i$, $\rho_i$, $e_i$, $\varepsilon_i$, $S_i$ are the elastic, density, piezoelectric, dielectric constants and bulk waves velocities.

Propagation of the interconnected elastic and electromagnetic excitations in a considered layered piezoelectric structure are described by the following system of motion and Maxwell's equations [1, 20, 21]:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = \rho \frac{\partial^2 u_i}{\partial t^2},$$

$$\varepsilon_{ijk}\frac{\partial H_k}{\partial x_j} = \frac{\partial D_i}{\partial t}, \varepsilon_{ijk}\frac{\partial E_k}{\partial x_j} = -\frac{\partial B_i}{\partial t}, \tag{1.1}$$

$$\frac{\partial B_i}{\partial x_i} = 0, \quad \frac{\partial D_i}{\partial x_i} = 0 \quad (i,j,k = 1,2,3),$$

where $u_i$ are the components of the displacement, $\sigma_{ij}$ are the components of the stress tensor, $H_k$ and $B_i$ are the magnetic field intensity and induction, $D_i$ and $E_k$ are the electric displacement and electric field intensity, $F_i$ are the densities of the bulk forces, $\varepsilon_{ijk}$ are the components of the Levy–Chivita tensor. It is assumed that the conducting current and free charges are not present and the piezoelectric media is none magnetic.

The constitutive equations are:

$$\sigma_{ij} = c_{ijmn}\gamma_{mn} - e_{kij}E_k,$$

$$D_i = \varepsilon_{ij}E_j + e_{ijk}\gamma_{jk}, \tag{1.2}$$

$$B_i = \mu_{ik}H_k.$$

Here $\gamma_{ij}$ are the components of a strain tensor:

$$\gamma_{jk} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j}\right), \tag{1.3}$$

$c_{ijmn}$, $e_{ijk}$, $\varepsilon_{ij}$, $\mu_{ij}$ $(i,j,k,m,n = 1,2,3)$ are the elastic, piezoelectric, dielectric and magnetic constants.

We consider an anti-plane deformation conditions which can be described by the following relations:

$$\mathbf{u} = (0,0,u_3), \quad u_3 = u_3(x_1,x_2,t),$$

$$\mathbf{H} = (0,0,H_3), \quad H_3 = H_3(x_1,x_2,t), \tag{1.4}$$

$$\mathbf{E} = (E_1,E_2,0), \quad E_i = E_i(x_1,x_2,t)$$

$$(i = 1,2).$$

Equation of medium motion and Maxwell's equations for electromagnetic fields under conditions (1.4) take the form:

$$c_{44}\nabla^2 u_3 - e_{15}\left(\frac{\partial E_1}{\partial x_1} + \frac{\partial E_2}{\partial x_2}\right) = \rho\frac{\partial^2 u_3}{\partial t^2},$$

$$e_{15}\nabla^2 u_3 + \varepsilon_{11}\left(\frac{\partial E_1}{\partial x_1} + \frac{\partial E_2}{\partial x_2}\right) = 0,$$

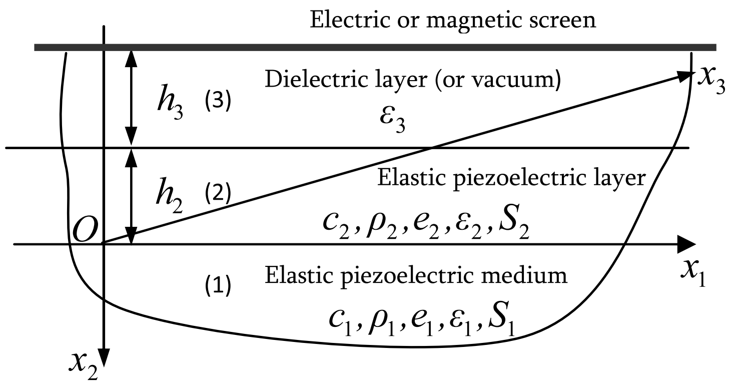


Figure 1. Piezoelectric layered medium in the presence of an electric screen

$$\begin{gathered}\frac{\partial E_2}{\partial x_1}-\frac{\partial E_1}{\partial x_2}=-\mu_{33}\frac{\partial H_3}{\partial t},\\ \frac{\partial H_3}{\partial x_2}=e_{15}\frac{\partial^2 u_3}{\partial x_1\partial t}+\varepsilon_{11}\frac{\partial E_1}{\partial t},\\ \frac{\partial H_3}{\partial x_1}=-e_{15}\frac{\partial^2 u_3}{\partial x_2\partial t}-\varepsilon_{11}\frac{\partial E_2}{\partial t},\\ \nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}.\end{gathered}\tag{1.5}$$

From (1.5) for the substrate and the layer we come to equations:

$$\nabla^2 u_3^{(i)}=\frac{1}{S_i^2}\frac{\partial^2 u_3^{(i)}}{\partial t^2}\quad (i=1,2)\,. \tag{1.6}$$

Here

$$S_i^2=\bar{c}_i/\rho_i,\quad \bar{c}_i=c_i\left(1+\chi_i^2\right),\quad \chi_i^2=e_i^2/(\varepsilon_i c_i),$$
$$c_i=c_{44}^{(i)},\quad e_i=e_{15}^{(i)},\quad \varepsilon_i=\varepsilon_{11}^{(i)};$$

$\bar{c}_i$ are the piezoelectrtically stiffened elastic constants, $S_i$ are the velocities of electro-magneto-elastic bulk waves in direction of the $Ox_1$ axis, $\chi_i$ are electro-mechanical coupling coefficients. The magnetic fields intensities both in the substrate and the layer satisfy the following equations:

$$\begin{gathered}\nabla^2 H_3^{(i)}=\frac{1}{a_i^2}\frac{\partial^2 H_3^{(i)}}{\partial t^2},\\ a_i^2=\frac{1}{\varepsilon_i\mu_i},\quad \mu_i=\mu_{33}^{(i)}\quad (i=1,2)\,,\end{gathered}\tag{1.7}$$

$H_3^{(i)}$ are the magnetic field intensities in the substrate and in the layer, $a_i$ are the velocities of the electro-magnetic bulk waves in the considered medium.

In the dielectric (or vacuum) layer the electro-magnetic field will be derived from the following Maxwell's equations:

$$\begin{gathered}\frac{\partial E_2^{(3)}}{\partial x_1}-\frac{\partial E_1^{(3)}}{\partial x_2}=-\mu_3\frac{\partial H_3^{(3)}}{\partial t},\\ \frac{\partial H_3^{(3)}}{\partial x_2}=\varepsilon_3\frac{\partial E_1^{(3)}}{\partial t},\\ \frac{\partial H_3^{(3)}}{\partial x_1}=-\varepsilon_3\frac{\partial E_2^{(3)}}{\partial t}.\end{gathered}\tag{1.8}$$

From (1.8) we have:

$$\nabla^2 H_3^{(3)}=\frac{1}{a_3^2}\frac{\partial^2 H_3^{(3)}}{\partial t^2},\quad a_3^2=\frac{1}{\varepsilon_3\mu_3}. \tag{1.9}$$

Here $a_3$ is the velocity of the electro-magnetic wave in dielectric layer, $\varepsilon_3$ and $\mu_3$ are dielectric and magnetic constants in dielectric medium.

### *1.1. The boundary and contact conditions*

On the plane at $x_2=0$ the continuity conditions applied for the displacement, tangential components of the electrical field intensity, the stress and the normal component of the electric displacement are:

$$\begin{gathered}u_3^{(1)}=u_3^{(2)},\quad E_1^{(1)}=E_1^{(2)},\\ \sigma_{23}^{(1)}=\sigma_{23}^{(2)},\quad D_2^{(1)}=D_2^{(2)}.\end{gathered}\tag{1.10}$$

At $x_2=-h_2$ continuity conditions are applied for tangential components of the electrical field intensity, the normal component of the electric displacement. The tangential component of the stress vanishes, all that will take the follows form:

$$E_1^{(2)}=E_1^{(3)},\quad \sigma_{23}^{(2)}=0,\quad D_2^{(2)}=D_2^{(3)}. \tag{1.11}$$

The last conditions in (1.10) and (1.11) can be replaced by the continuity condition of the tangential component of the magnetic field intensity:

$$H_3^{(1)} = H_3^{(2)} \text{ at } x_2 = 0 \text{ and}$$

$$H_3^{(2)} = H_3^{(3)} \text{ at } x_2 = -h_2.$$

The equivalence of the boundary conditions $D_2^{(1)} = D_2^{(2)}$ and $H_3^{(1)} = H_3^{(2)}$ at $x_2 = 0$ have been first noticed by Belubekyan [6] and it follows that in quasistatic approximation the boundary condition $D_2 = 0$ at $x_2 = 0$ is the boundary condition for the magnetic field screen.

The boundary condition on the electric field screen (at$x_2 = -(h_2 + h_3)$) for tangential component of the electric field intensity is:

$$E_1^{(3)} = 0. \tag{1.12}$$

In the case of magnetic field screen the magnetic field intensity also vanishes:

$$H_3^{(3)} = 0. \tag{1.13}$$

This condition is equivalent to the following condition:

$$D_2^{(3)} = 0. \tag{1.14}$$

Hence the condition (1.12) in quasistatic approximation corresponds to the electric field screen, but the condition (1.14) corresponds to the magnetic field screen.

Thus the dynamic approximation enables us to explain the condition on the screen in the case of quasistatic approximation.

*1.2. The attenuation conditions*

The solutions $u_3^{(1)}$ and $H_1^{(3)}$ must attenuate in the half space substrate:

$$\lim_{x_2\to+\infty} u_3^{(1)} = 0, \quad \lim_{x_2\to+\infty} H_3^{(1)} = 0.$$

In particular case when the screen is in the infinity the attenuation of the function $H_3^{(3)}$ is as follows:

$$\lim_{x_2\to-\infty} H_3^{(3)} = 0, \tag{1.15}$$

it is in accordance with the condition (1.13) and it can be shown that it is equivalent to the condition (1.12) also if $x_2 \to -\infty$. Thus when the screen is at the infinity the conditions (1.12) and (1.13) are equivalent to the condition (1.15).

The statement of the problem is to find out and investigate the behavior of the elastic and electro-magnetic fields in considered piezoelectric structures in fully dynamic settings, taking into account boundary, contact and attenuation conditions, and to find out and analyze the dispersion equation.

## 2. Solutions of the problem as a plane waves

**a) Solutions in the piezoelectric substrate**. We seek the solutions of equations (1.6) and (1.7) as plane harmonic waves:

$$\begin{aligned} u_3^{(1)} &= \tilde{w}_1 e^{i(qx_2+px_1-\omega t)}, \\ H_3^{(1)} &= \tilde{H}_1 e^{i(qx_2+px_1-\omega t)}. \end{aligned} \tag{2.1}$$

Here $p > 0$, $q$ are the horizontal and shear waves numbers, $\omega > 0$ is the circle frequency and $\tilde{w}_1$, $\tilde{H}_1$ are the amplitudes. Substituting (2.1) in (1.6), (1.7) and satisfying to none zero wave numbers existence condition we obtain the following dispersion equations:

$$q^2 = -p^2 + \frac{\omega^2}{S_1^2}, \quad q^2 = -p^2 + \frac{\omega^2}{a_1^2}. \tag{2.2}$$

From the first equation of (2.2) we have:

$$q = \pm ip\beta_1(V), \quad \beta_1(V) = \sqrt{1 - \frac{V^2}{S_1^2}}, \tag{2.3}$$

$V = \omega/p > 0$ is the phase velocity of the seeking surface wave.

From the second equation of (2.2) we obtain:

$$q = \pm ip\gamma_1(V), \quad \gamma_1(V) = \sqrt{1 - \frac{V^2}{a_1^2}}. \tag{2.4}$$

The attenuation conditions at $x_2 \to +\infty$ are:

$$\beta_1(V) > 0, \quad \gamma_1(V) > 0, \tag{2.5}$$

with the plus sign in the formulas (2.3) and (2.4). The corresponding solutions take the following form:

$$\begin{aligned} u_3^{(1)} &= \tilde{w}_1^- e^{-p\beta_1 x_2} e^{i(px_1-\omega t)}, \\ H_3^{(1)} &= \tilde{H}_1^- e^{-p\gamma_1 x_2} e^{i(px_1-\omega t)}. \end{aligned} \tag{2.6}$$

From (2.5) follows:

$$V < S_1 < a_1 \tag{2.7}$$

Hence if the surface wave exists its phase velocity is not bigger than the velocity of a bulk electro-elastic waves in the substrate. Thus the wave fields in piezoelectric substrate are described by functions (2.6), where $\tilde{w}_1^-$ and $\tilde{H}_1$ are unknown constants and $\beta_1(V) > 0$, $\gamma_1(V) > 0$ are defined by formulas (2.3) and (2.4). In the substrate phase velocities satisfy to conditions (2.5) and (2.7).

**b) Solutions in the piezoelectric layer.** In the same way we can find the solutions in the piezoelectric layer, which will take the following form:

$$u_3^{(2)} = \left[\tilde{w}_2^- e^{-p\beta_2 x_2} + \tilde{w}_2^+ e^{p\beta_2 x_2}\right] \times e^{i(px_1-\omega t)}, \quad (2.8)$$

$$H_3^{(2)} = \left[\tilde{H}_2^- e^{-p\gamma_2 x_2} + \tilde{H}_2^+ e^{p\gamma_2 x_2}\right] \times e^{i(px_1-\omega t)}, \quad (2.9)$$

here:

$$\beta_2(V) = \sqrt{\frac{V^2}{S_2^2} - 1}, \qquad \gamma_2(V) = \sqrt{1 - \frac{V^2}{a_2^2}}. \quad (2.10)$$

If $V < a_2$ then $\gamma_2(V) > 0$. The function $\beta_2(V)$ can be both real positive and imaginary, it is real for $V > S_2$. In that case homogeneous electro-elastic waves propagate through the layer undergoing full internal reflection from layer boundaries (as in the case of the classical Love waves). If $V < S_2$ the function $\beta_2(V)$ is imaginary then through the layer will propagate inhomogeneous waves creating so called gap waves (in classical Love problem there are no such kind of waves). The waves of the magnetic field are always inhomogeneous. Hence in the layer we should take the solutions according both to the positive and negative roots:

$$\begin{aligned} u_3^{(2)} &= \left[\tilde{w}_2^- e^{-ip\beta_2 x_2} + \tilde{w}_2^+ e^{ip\beta_2 x_2}\right] \times e^{i(px_1-\omega t)}, \\ H_3^{(2)} &= \left[\tilde{H}_2^- e^{-ip\gamma_2 x_2} + \tilde{H}_2^+ e^{ip\gamma_2 x_2}\right] \times e^{i(px_1-\omega t)}. \end{aligned} \quad (2.11)$$

**c) The solutions in the dielectric layer (or vacuum).** The solutions in dielectric layer we'll seek in the following form:

$$H_3^{(3)} = \tilde{H}_3 e^{i(qx_2+px_1-\omega t)}. \quad (2.12)$$

In analogous way we obtain the dispersion equation:

$$q^2 = -p^2 + \frac{\omega^2}{a_3^2},$$

then we have:

$$q = \pm ip\gamma_3(V), \quad \gamma_3(V) = \sqrt{1 - \frac{V^2}{a_3^2}}, \quad (2.13)$$

if $V < a_3$ then $\gamma_3(V) > 0$. Thus the solution is the combination of those solutions with the wave numbers as in (2.13):

$$H_3^{(3)} = \left[\tilde{H}_3^- e^{-p\gamma_3 x_2} + \tilde{H}_3^+ e^{p\gamma_3 x_2}\right] \times e^{i(px_1-\omega t)}. \quad (2.14)$$

Hence we find the solution of the problem as the sum of constituent solutions (2.6), (2.11) and (2.14) in the substrate, the layer and dielectric medium accordingly.

## 3. The contact and boundary conditions transformation

**a) The determination of the functions $\boldsymbol{E_1^{(1)}}$, $\boldsymbol{E_2^{(1)}}$, $\boldsymbol{\sigma_{23}^{(1)}}$, $\boldsymbol{D_2^{(1)}}$ in the piezoelectric substrate.** From the fourth equation of the system (1.5) can be found $\partial E_1^{(1)}/\partial t$ and taking into account the solutions in the substrate (2.6) we obtain:

$$\frac{\partial E_1^{(1)}}{\partial t} = -\frac{p}{\varepsilon_1}\left[\gamma_1 \tilde{H}_1^- e^{-p\gamma_1 x_2} + e_1\omega \tilde{w}_1^- e^{-p\beta_1 x_2}\right] \times e^{i(px_1-\omega t)}. \quad (3.1)$$

Since $E_1^{(1)} = \tilde{E}_1(x_2)e^{i(px_1-\omega t)}$ then

$$\frac{\partial E_1^{(1)}}{\partial t} = -i\omega E_1^{(1)}. \quad (3.2)$$

From (3.1) and (3.2) it follows:

$$E_1^{(1)} = t - \frac{ip}{\varepsilon_1 \omega}\left[\gamma_1 \tilde{H}_1^- e^{-p\gamma_1 x_2} + e_1\omega \tilde{w}_1^- e^{-p\beta_1 x_2}\right] \times e^{i(px_1-\omega t)}. \quad (3.3)$$

In analogous way from the fifth equation of (1.5) and (2.6) we have:

$$E_2^{(1)} = -\frac{p}{\varepsilon_1\omega}\left[\tilde{H}_1^- e^{-p\gamma_1 x_2} + e_1\beta_1\omega\tilde{w}_1^- e^{-p\beta_1 x_2}\right] \times e^{i(px_1-\omega t)}. \quad (3.4)$$

The second equation (1.5) yields:

$$\sigma_{23}^{(1)} = c_1\frac{\partial u_3^{(1)}}{\partial x_2} - e_1 E_2^{(1)}. \quad (3.5)$$

From (2.7), (3.4) and (3.5) we have:

$$\sigma_{23}^{(1)} = -p\left[\bar{c}_1\beta_1\tilde{w}_1^- e^{-p\beta_1 x_2} + \frac{e_1}{\varepsilon_1\omega}H_1^- e^{-p\gamma_1 x_2}\right] \times e^{i(px_1-\omega t)}. \quad (3.6)$$

Finally substituting $E_3^{(1)}$ and $E_2^{(1)}$ into the fourth equation of (1.5) we obtain:

$$D_2^{(1)} = \frac{p}{\omega}H_1^- e^{-p\gamma_1 x_2}e^{i(px_1-\omega t)}. \quad (3.7)$$

**b) The determination of the functions $\boldsymbol{E_1^{(2)}}$, $\boldsymbol{E_2^{(2)}}$, $\boldsymbol{\sigma_{23}^{(2)}}$, $\boldsymbol{D_2^{(2)}}$ in the piezoelectric layer.** From (1.5), (1.6) and (2.11) the following expressions will hold:

$$E_1^{(2)} = -\frac{ip}{\varepsilon_2\omega}\left[\gamma_2\left(\tilde{H}_2^- e^{-p\gamma_2 x_2} - \tilde{H}_2^+ e^{p\gamma_2 x_2}\right) + e_2\omega\left(\tilde{w}_2^- e^{-ip\beta_2 x_2} + \tilde{w}_2^+ e^{ip\beta_2 x_2}\right)\right]e^{i(px_1-\omega t)},$$

$$E_2^{(2)} = \frac{p}{\varepsilon_2\omega}\left[\left(\tilde{H}_2^- e^{-p\gamma_2 x_2} + \tilde{H}_2^+ e^{p\gamma_2 x_2}\right) + ie_2\omega\beta_2\left(\tilde{w}_2^- e^{-ip\beta_2 x_2} - \tilde{w}_2^+ e^{ip\beta_2 x_2}\right)\right]e^{i(px_1-\omega t)},$$

$$\sigma_{23}^{(2)} = -ip\left[-\frac{ie_2}{\omega\varepsilon_2}\left(\tilde{H}_2^- e^{-p\gamma_2 x_2} + \tilde{H}_2^+ e^{p\gamma_2 x_2}\right) + \bar{c}_2\beta_2\left(\tilde{w}_2^- e^{-ip\beta_2 x_2} - \tilde{w}_2^+ e^{ip\beta_2 x_2}\right)\right]e^{i(px_1-\omega t)}, \quad (3.8)$$

$$D_2^{(2)} = \frac{p}{\omega}\left[\tilde{H}_2^- e^{-p\gamma_2 x_2} + \tilde{H}_2^+ e^{p\gamma_2 x_2}\right]e^{i(px_1-\omega t)}.$$

**c) The boundary conditions at $\boldsymbol{x_2 = 0}$.** From (2.6), (2.8), (3.3), (3.6), (3.7) and (3.8) we obtain:

$$\begin{aligned}
&\tilde{w}_1^- = \tilde{w}_2^- + \tilde{w}_2^+,\\
&\frac{\gamma_1}{\varepsilon_1}\tilde{H}_1^- + \bar{e}_1\omega\tilde{w}_1^- = \frac{\gamma_2}{\varepsilon_2}\left(\tilde{H}_2^- - \tilde{H}_2^+\right) + \bar{e}_2\omega\left(\tilde{w}_2^- + \tilde{w}_2^+\right),\\
&\frac{\bar{e}_1}{\omega}\tilde{H}_1^- + \bar{c}_1\beta_1\tilde{w}_1^- = \frac{\bar{e}_2}{\omega}\left(\tilde{H}_2^- - \tilde{H}_2^+\right) + i\bar{c}_2\beta_2\left(\tilde{w}_2^- - \tilde{w}_2^+\right),\\
&\tilde{H}_1^- = \tilde{H}_2^- + \tilde{H}_2^+,
\end{aligned} \quad (3.9)$$

here $\bar{e}_1 = e_1/\varepsilon_1$, $\bar{e}_2 = e_2/\varepsilon_2$.

**d) The determination of the functions $\boldsymbol{E_1^{(3)}}$, $\boldsymbol{E_2^{(3)}}$ and $\boldsymbol{D_2^{(3)}}$ in the dielectric layer.** Substituting solutions (2.14) into last two equations of (1.9) we have:

$$\begin{aligned}
E_1^{(3)} &= -\frac{ip\gamma_3}{\varepsilon_3\omega}\left[\tilde{H}_3^- e^{-p\gamma_3 x_2} - \tilde{H}_3^+ e^{p\gamma_3 x_2}\right] \times e^{i(px_1-\omega t)},\\
E_2^{(3)} &= \frac{p}{\varepsilon_3\omega}\left[\tilde{H}_3^- e^{-p\gamma_3 x_2} + \tilde{H}_3^+ e^{p\gamma_3 x_2}\right] \times e^{i(px_1-\omega t)}.
\end{aligned} \quad (3.10)$$

The connection between the electric field intensity and the electric displacement is:

$$D_1^{(3)} = \varepsilon_3 E_1^{(3)}, \quad D_2^{(3)} = \varepsilon_3 E_2^{(3)}.$$

Taking into account (3.10) we have:

$$D_2^{(3)} = \frac{p}{\omega}\left[\tilde{H}_3^- e^{-p\gamma_3 x_2} + \tilde{H}_3^+ e^{p\gamma_3 x_2}\right]. \quad (3.11)$$

**e) The boundary conditions at $\boldsymbol{x_2 = -h_2}$.** Substituting (3.8), (3.10) and (3.11) in boundary conditions (1.11) the following expressions will hold:

$$\frac{\gamma_2}{\varepsilon_2}\left[\tilde{H}_2^- e^{\gamma_2 k_2} - \tilde{H}_2^+ e^{-\gamma_2 k_2}\right] + \bar{e}_2\omega\left[\tilde{w}_2^- e^{i\beta_2 k_2} + \tilde{w}_2^+ e^{-i\beta_2 k_2}\right] = \frac{\gamma_3}{\varepsilon_3}\left[\tilde{H}_3^- e^{\gamma_3 k_2} - \tilde{H}_3^+ e^{-\gamma_3 k_2}\right],$$

$$\frac{\bar{e}_2}{\omega}\left[\tilde{H}_2^- e^{\gamma_2 k_2} + \tilde{H}_2^+ e^{-\gamma_2 k_2}\right] + i\bar{c}_2\beta_2\left[\tilde{w}_2^- e^{i\beta_2 k_2} - \tilde{w}_2^+ e^{-i\beta_2 k_2}\right] = 0, \quad (3.12)$$

$$\tilde{H}_2^- e^{\gamma_2 k_2} + \tilde{H}_2^+ e^{-\gamma_2 k_2} = \tilde{H}_3^- e^{\gamma_3 k_2} + \tilde{H}_3^+ e^{-\gamma_3 k_2}.$$

Here $k_2 = ph_2$ is the relative thickness of the piezoelectric layer.

**f) The boundary conditions on the screen (at $x_2 = -(h_2 + h_3)$).**

*The case of the electric screen.* After substituting $E_1^{(3)}$ from (3.10) into the boundary conditions (1.12) we obtain:

$$\tilde{H}_3^- e^{\gamma_3(k_2+k_3)} - \tilde{H}_3^+ e^{-\gamma_3(k_2+k_3)} = 0, \quad (3.13)$$

here $k_3 = ph_3$ is the relative thickness of the dielectric medium.

*The case of the magnetic screen.* Substituting $H_3^{(3)}$ from (2.14) into the boundary condition (1.13) we obtain:

$$\tilde{H}_3^- e^{\gamma_3(k_2+k_3)} + \tilde{H}_3^+ e^{-\gamma_3(k_2+k_3)} = 0. \quad (3.14)$$

The same condition we'll have substituting $D_2^{(3)}$ from (3.11) into the boundary condition (1.14). Thus we come to conclusion that in the case of electric field screen we need to use the boundary condition (3.13) and in the case of magnetic field screen the condition (3.14).

**g) The contact and the boundary conditions transformation in the case of electric field screen.** Let's substitute $\tilde{H}_1^-$ and $\tilde{w}_1^-$ from the first and the fourth equations of (3.9) into the second and the third equations of (3.9). Then from the third equation of (3.12) and from the condition (3.13) we define $\tilde{H}_3^-$ and $\tilde{H}_3^+$ which can be expressed through $\tilde{H}_2^-$ and $\tilde{H}_2^+$ in the following way:

$$\begin{aligned} \tilde{H}_3^- &= \frac{e^{-\gamma_3 k_2}}{1+e^{2\gamma_3 k_3}} \times(\tilde{H}_2^- e^{\gamma_2 k_2} + \tilde{H}_2^+ e^{-\gamma_2 k_2}), \\ \tilde{H}_3^+ &= \frac{e^{\gamma_3(k_2+2k_3)}}{1+e^{2\gamma_3 k_3}} \times(\tilde{H}_2^- e^{\gamma_2 k_2} + \tilde{H}_2^+ e^{-\gamma_2 k_2}). \end{aligned} \quad (3.15)$$

## 4. The determination of the dispersion equation

After some transformations of relations (3.9), (3.12), (3.13) and (3.15), we obtain the following homogeneous algebraic equations system for the undetermined amplitudes $\tilde{w}_2^-$, $\tilde{w}_2^+$, $\tilde{H}_2^-$, $\tilde{H}_2^+$:

$$\begin{aligned}
&(\bar{c}_1\beta_1 - i\bar{c}_2\beta_2)\,\tilde{w}_2^- + (\bar{c}_1\beta_1 + i\bar{c}_2\beta_2)\,\tilde{w}_2^+ + \frac{\bar{e}_1-\bar{e}_2}{\omega}\tilde{H}_2^- + \frac{\bar{e}_1-\bar{e}_2}{\omega}\tilde{H}_2^+ = 0,\\
&\omega(\bar{e}_1-\bar{e}_2)\,\tilde{w}_2^- + \omega(\bar{e}_1-\bar{e}_2)\,\tilde{w}_2^+ + \left(\frac{\gamma_1}{\varepsilon_1} - \frac{\gamma_2}{\varepsilon_2}\right)\tilde{H}_2^- + \left(\frac{\gamma_1}{\varepsilon_1} + \frac{\gamma_2}{\varepsilon_2}\right)\tilde{H}_2^+ = 0,\\
&i\bar{c}_2\beta_2 e^{ik_2\beta_2}\tilde{w}_2^- - i\bar{c}_2\beta_2 e^{-ik_2\beta_2}\tilde{w}_2^+ + \frac{\bar{e}_2}{\omega}e^{k_2\gamma_2}\tilde{H}_2^- + \frac{\bar{e}_2}{\omega}e^{-k_2\gamma_2}\tilde{H}_2^+ = 0,\\
&\omega\bar{e}_2 e^{ik_2\beta_2}\tilde{w}_2^- + \omega\bar{e}_2 e^{-ik_2\beta_2}\tilde{w}_2^+ + \left(\frac{\gamma_2}{\varepsilon_2} + \frac{\gamma_3}{\varepsilon_3}\delta_3\right)e^{k_2\gamma_2}\tilde{H}_2^- - \left(\frac{\gamma_2}{\varepsilon_2} - \frac{\gamma_3}{\varepsilon_3}\delta_3\right)e^{-k_2\gamma_2}\tilde{H}_2^+ = 0,
\end{aligned} \quad (4.1)$$

here $\delta_3 = \tanh(k_3\gamma_3)$. The existence condition of none zero solution of (4.1) leads to the following dispersion equation:

$$\Psi = \{\psi_{ij}\} = 0, \quad i,j = 1,\ldots,4, \quad (4.2)$$

$$\psi_{11} = \bar{c}_1\beta_1 - i\bar{c}_2\beta_2, \quad \psi_{12} = \bar{c}_1\beta_1 + i\bar{c}_2\beta_2,$$

$$\psi_{13} = \psi_{14} = \frac{\bar{e}_1-\bar{e}_2}{\omega},$$

$$\psi_{21} = \psi_{22} = \omega(\bar{e}_1-\bar{e}_2),$$

$$\psi_{23} = \frac{\gamma_1}{\varepsilon_1} - \frac{\gamma_2}{\varepsilon_2}, \quad \psi_{24} = \frac{\gamma_1}{\varepsilon_1} + \frac{\gamma_2}{\varepsilon_2},$$

$$\psi_{31} = i\bar{c}_2\beta_2 e^{ik_2\beta_2}, \quad \psi_{32} = i\bar{c}_2\beta_2 e^{-ik_2\beta_2},$$

$$\psi_{33} = \frac{\bar{e}_2}{\omega}e^{k_2\gamma_2}, \quad \psi_{34} = \frac{\bar{e}_2}{\omega}e^{-k_2\gamma_2},$$

$$\psi_{41} = \omega\bar{e}_2 e^{ik_2\beta_2}, \quad \psi_{42} = \omega\bar{e}_2 e^{-ik_2\beta_2},$$

$$\psi_{43} = \left(\frac{\gamma_2}{\varepsilon_2} + \frac{\gamma_3}{\varepsilon_3}\delta_3\right)e^{k_2\gamma_2},$$

$$\psi_{44} = -\left(\frac{\gamma_2}{\varepsilon_2} - \frac{\gamma_3}{\varepsilon_3}\delta_3\right)e^{-k_2\gamma_2}.$$

This equation can be written in the following form:

$$A(k_2,k_3,V)\sin(k_2\beta_2) + B(k_2,k_3,V)\cos(k_2\beta_2) = E(V), \quad (4.3)$$

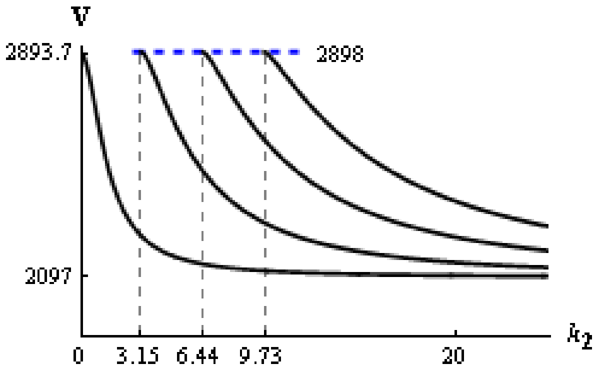


Figure 2. Dependence $V$ on $k_2$, soft layer, the screen is at a finite distance from the piezoelectric layer: $\delta_3 = 0.1$, $V_{BG} = 2893.7$.

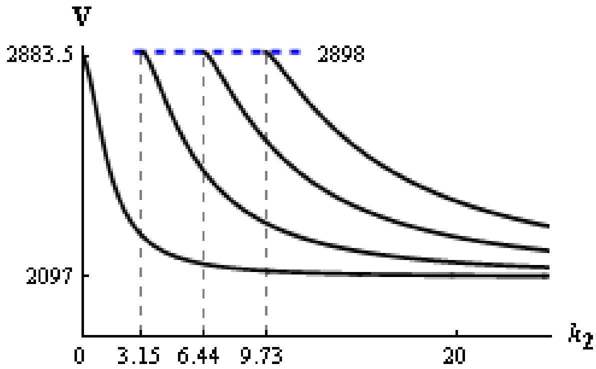


Figure 3. Dependence $V$ on $k_2$, soft layer, the screen is located on a surface of the piezoelectric layer: $\delta_3 = 0$, $V_{BG} = 2883.5$.

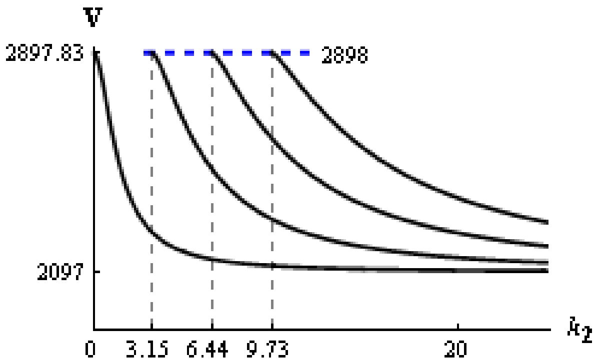


Figure 4. Dependence $V$ on $k_2$, soft layer, screen is at infinity: $\delta_3 = 1$, $V_{BG} = 2897.83$.

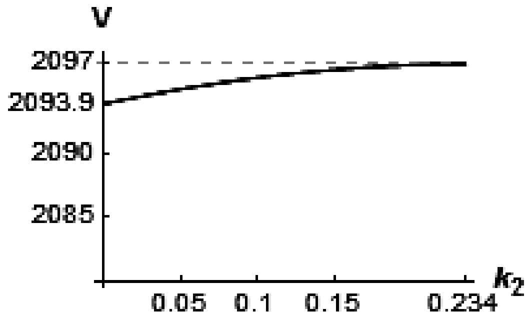


Figure 5. Dependence $V$ on $k_2$, hard layer, the screen is at a finite distance from the piezoelectric layer: $\delta_3 = 0.1$, $V_{BG} = 2093.9$.

here:

$$\begin{aligned} A(k_2,k_3,V) &= \mathrm{ch}\,(k_2\gamma_2)\,\gamma_2\varepsilon_2 \\ &\times \left[\beta_2^2\bar{c}_2^2\left(\gamma_3\delta_3\varepsilon_1+\gamma_1\varepsilon_3\right)-\beta_1\varepsilon_1\varepsilon_3\bar{c}_1\bar{e}_2^2\right] \\ &+ \mathrm{sh}\,(k_2\gamma_2)\Big[\beta_2^2\bar{c}_2^2\left(\gamma_1\gamma_3\delta_3\varepsilon_2^2+\gamma_2^2\varepsilon_1\varepsilon_3\right) \\ &+ \varepsilon_2^2\varepsilon_3\bar{e}_2^2\Big(\varepsilon_1\left(\bar{e}_1-\bar{e}_2\right)^2-\beta_1\gamma_1\bar{c}_1\Big)\Big]; \end{aligned}$$

$$\begin{aligned} B(k_2,k_3,V) &= \mathrm{ch}\,(k_2\gamma_2)\,\beta_2\bar{c}_2\gamma_2\varepsilon_2 \\ &\times\left[\varepsilon_1\varepsilon_3\left(\bar{e}_1^2-2\bar{e}_1\bar{e}_2+2\bar{e}_2^2\right)-\bar{c}_1\beta_1\left(\gamma_3\delta_3\varepsilon_1+\gamma_1\varepsilon_3\right)\right] \\ &+\mathrm{sh}\,(k_2\gamma_2)\,\beta_2\bar{c}_2\Big[\varepsilon_2^2\Big(\gamma_3\delta_3\varepsilon_1\left(\bar{e}_1-\bar{e}_2\right)^2+\bar{e}_2^2\gamma_1\varepsilon_3\Big) \\ &-\beta_1\bar{c}_1\left(\gamma_1\gamma_3\delta_3\varepsilon_2^2+\gamma_2^2\varepsilon_1\varepsilon_3\right)\Big]; \quad (4.4) \end{aligned}$$

$$E(V) = -2\bar{c}_2\bar{e}_2\gamma_2\varepsilon_1\varepsilon_2\varepsilon_3\left(\bar{e}_1-\bar{e}_2\right)\beta_2\cos^2\left(k_2\beta_2\right).$$

Below the dispersion curves are represented (see fig. 2–7) for two structures: first of it consists of the zinc oxide (ZnO) substrate (parameters are [22]: $\rho_1 = 5.7\times10^3$ kg/m$^3$, $S_1 = 2898$ m/sec, $c_1 = 4.25\times10^{10}$ N/m$^2$, $\varepsilon_1 = 7.38\times10^{-11}$ F/m, $e_1 = -0.59$ C/m$^2$), paratellurite (TeO2) layer (parameters are [22]: $\rho_2 = 6.0\times10^3$ kg/m$^3$, $S_2 = 2097$ m/sec, $c_2 = 2.65\times10^{10}$ N/m$^2$, $\varepsilon_2 = 20.0\times10^{-11}$ F/m, $e_2 = 0$) (soft layer) and the dielectric medium which is a vacuum ($\varepsilon_3 = 8.85\times10^{-12}$), the other one vice versa: the layer is a zinc oxide (ZnO), the substrate is a paratellurite (hard layer). For each of them three various cases of the electric screen locations are investigated:

a) The screen is at a finite distance of $h_3$ from the layer ($\delta_3 = 0.1$).

b) The screen is in infinity ($\delta_3 = 1$).

c) The screen is located on a surface of the piezoelectric layer ($\delta_3 = 0$), it means that there is no dielectric medium.

In Figures 2–5 for the soft layer case exist many modes of surface waves, which as an asymptote for $k_2\to\infty$ has a line $V = S_2$. The first mode for $k_2 = 0$ occurs when $V = V_{BG}$.

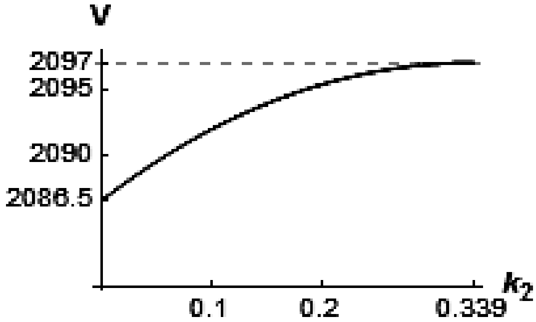


Figure 6. Dependence $V$ on $k_2$, hard layer, the screen is located on a surface of the piezoelectric layer: $\delta_3 = 0$, $V_{BG} = 2086.5$

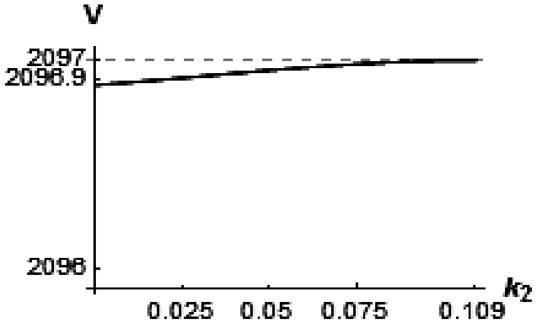


Figure 7. Dependence $V$ on $k_2$, hard layer, screen is at infinity: $\delta_3 = 1$, $V_{BG} = 2096.88$

In Figures 5–7 for the hard layer case a single curve is shown that reaches its maximum certain value $k_2$, after which there is no wave process.

## Conclusion

1. The existence and behavior of surface electro-magneto-elastic waves in layered electro -elastic structures consisting of a piezoelectric substrate of crystal classes 6mm, 4mm, an elastic piezoelectric layer and an adjoining dielectric medium on the top with an electric (or magnetic) screen are considered. The existence of surface electro-magneto-elastic waves in layered electro-elastic structures and behavior of the modes of these waves are revealed.

2. In the case when the screen is located at a finite distance (see Fig. 2) from the layer and the case of soft layer, the dependence on different thicknesses of the piezoelectric layer, arise modes of electro-magneto-elastic waves with velocities decreasing to common limit such as velocity of the bulk wave in the layer $S_2$ and besides the first mode decreases from the velocity of the Bleustein-Gulyaev wave. Subsequent modes velocity decreases from the bulk wave velocity $S_1$ in the substrate. For the two other cases of screen location at infinity and at the surface of the layer (see Fig. 3, 4) all the modes velocities started decreasing from the Bleustein-Gulyaev wave velocity.

3. For the hard layer in each event of the electric screen location the waves are existed in a certain, finite interval of layer thicknesses (different intervals for each event of screen location), outside this interval there are no waves. Velocities of these waves are limited of bulk wave velocity in the substrate $S_1$.

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